CHAPTER 5

RESULTS & DISCUSSION

From the discussion held in previous chapters, it is evident that reducing the order of a system is emerging as a significant area to procure knowledge about the working of a system. Moreover, the modifications in the large systems are also achieved more effectively by reducing its order. There are numerous techniques developed to achieve the goal but having certain limitations. These limitations are overcome by developing a new technique IPCG, which applies to all types of systems. Moreover, the reduced denominator multinomial is formed by experiencing all poles of the large system. Also, the technique of genetic algorithm procures optimised results. The operating procedure of the IPCG algorithm is presented in the previous chapter. The newly formed combination of two techniques is applied to the continuous-time, discrete-time, SISO and MIMO examples to obtain its working capacity. The nine examples of the continuous-time SISO systems are reduced by the IPCG method taking the highest order of the original system as 48. Also, three examples of an order of 8^{th} of discrete-time type are reduced by the proposed method. For this, the value of errors is achieved lower than the literature techniques; hence it results in the increased accuracy of the obtained lower model. Moreover, the study of performance parameters reveals that the reduced model procured by the IPCG method approximates the system of higher-order more closely compared to the literature methods. Furthermore, the IPCG algorithm is applied for the reduction of MIMO systems. Three examples of continuous type MIMO systems up to 19^{th} order are considered for the reduction. Again, the reduced models are obtained with better accuracy and better performance than the literary techniques. For analysing the equivalency among the system of the higher and lower order, time-moments of both systems are also compared. The time-moments of both systems are procured having almost similar values. Moreover, the stability analysis reveals that the stability of the system of higher-order does not alter after reduction. Hence signifies that a system remains stable even after reducing its order to a lower value. The illustration of test examples subject to the reduction by IPCG is done as follows:

5.1 REDUCTION OF CONTINOUS TIME SISO SYSTEMS

***Example 1*:** A 2nd order reduced approximation is derived from a 4th order SISO system, which is the minimum phase. The 4th order system's transfer function (Gautam et al., 2019; Narwal & Prasad, 2017; Parmar et al., 2007b; Sikander & Prasad, 2015a, 2017; Tiwari & Kaur, 2020b; Vishwakarma & Prasad, 2008) is described as follows:

$$G_4(s) = \frac{s^3 + 7s^2 + 24s + 24}{s^4 + 10s^3 + 35s^2 + 50s + 24} \tag{5.1}$$

From the methodology, it is clear that the initial step for obtaining the approximation of reduced-order is to expand the equation by implying a partial fraction technique to get the poles and MDIs of the prescribed system. Post expansion, the system's poles are $\sigma_1 = -1$, $\sigma_2 = -2$, $\sigma_3 = -3$ and $\sigma_4 = -4$ and the MDI associated with each pole is $\eta_1 = 1$, $\eta_2 = 2$, $\eta_3 = 1$ and $\eta_4 = 1$. Now, the 4th order system is to be converted into the 2nd order approximated system. As described in the methodology section, two clusters will be formed from four poles of the given system of higher-order. According to MDI values, the first cluster is formed of only one pole $\sigma_1 = -1$ and hence forming the cluster centre $\sigma_{c1} = -1$ and second cluster consists of three poles $\sigma_2 = -2$, $\sigma_3 = -3$ and $\sigma_4 = -4$ and the second cluster's centre is procured as $\sigma_{c2} = -2.3844$ by solving the equation (4.9). After combining the two cluster centres, the 2nd order denominator equation of the system of the reduced order is formed and is described in equation (4.10).

$$D_2(s) = s^2 + 3.384s + 2.384$$

Now, the denominator equation of the system of reduced-order is utilised for obtaining the numerator of the same. Considering that the numerator is of 1st order having two unknown coefficients. Finally, the system of reduced-order is displayed as follows:

$$G_2(s) = \frac{m_1 s + m_2}{s^2 + 3.384s + 2.384} \tag{5.2}$$

The next step is to find the value of unknown coefficients by genetic algorithm (GA). The integral square error between the system of higher order's step response (5.1) and the approximated system of reduced-order (5.2) is minimised by GA, and the value of unknown coefficients m_1 and m_2 are obtained. A total 82 number of iterations are utilised to get the optimised results. After optimisation, the value of unknown coefficients is obtained as $m_1=0.7657$ and $m_2=2.384$. By putting the value of unknown coefficients, the equation for the system of reduced-order is procured through the following:

$$G_2(s) = \frac{0.7657s + 2.384}{s^2 + 3.384s + 2.384} \tag{5.3}$$

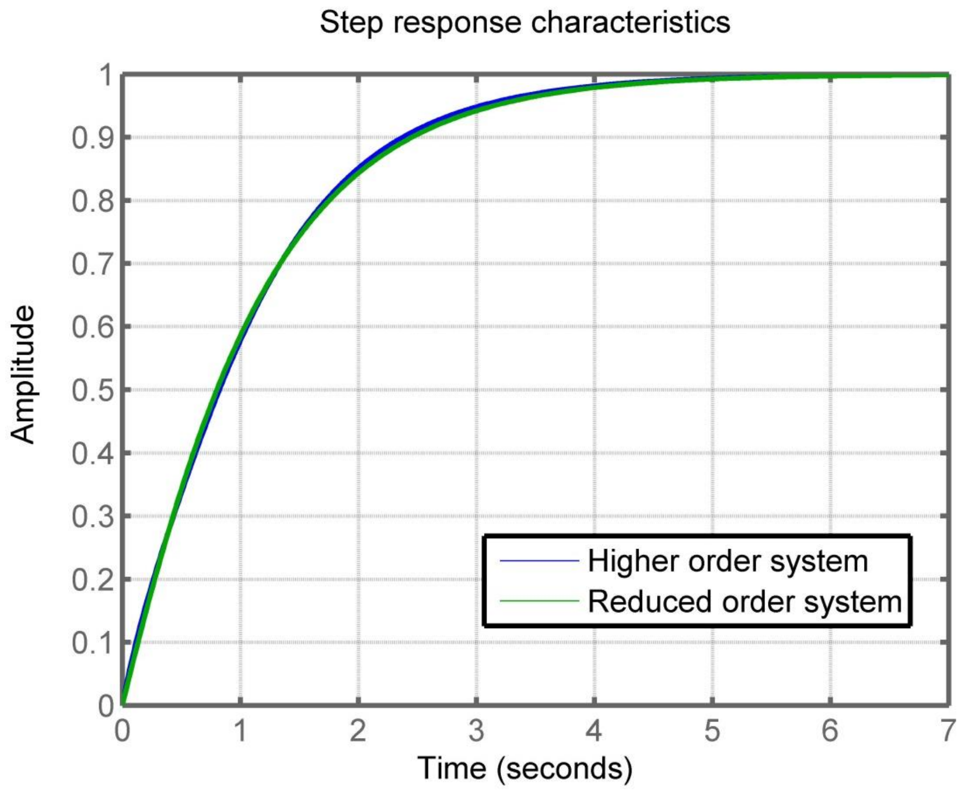


Figure 5.1. Step response characteristics of the system of higher and reduced-order for example 1

For checking the accuracy of the system of the reduced-order obtained in equation (5.3), it is compared with the system of higher-order based on the step and frequency response behaviour of a system of higher-order and of reduced-order. The step response behaviour is depicted in figure 5.1, which shows that both the systems of higher and reduced-order show the same behavioural characteristics. From Table 5.1, it is clear that the ISE, ITAE and IAE have shallow values, hence signifying that there is significantly less error in between the high and low order systems. Also, the rise time, settling time, overshoot and steady-state error's values for a unit step input is noted in Table 5.1. It shows that both systems' rising time and settling time values are almost the same, and the steady-state error between both systems is zero.

Table 5.1. Value of errors and various parameters to compare the higher system of 4th order and obtained system of 2nd order for a unit step input

Parameter	ISE	ITAE	IAE	OS (%)	RT (sec.)	ST (sec.)	SSE	G M	P M
HOS	-	-	-	0	2.26	3.93	-	∞	180
ROS	1.815 $\times 10^{-04}$	0.058	0.027	0	2.32	4.06	0	∞	180

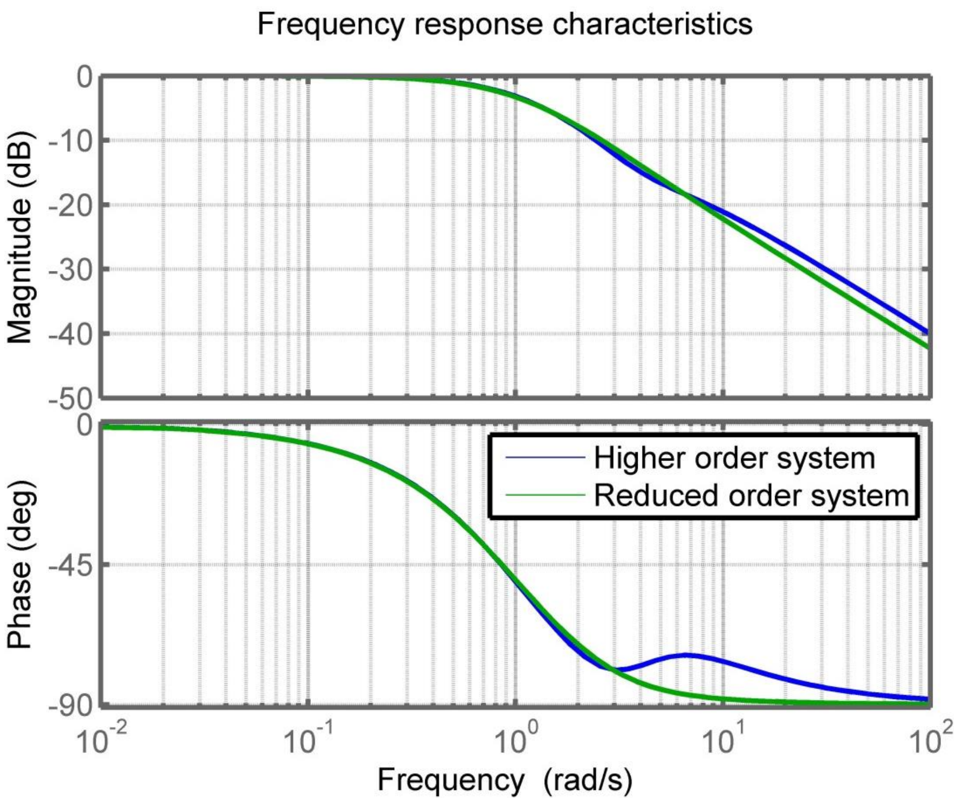


Figure 5.2. Example 1's frequency response characteristics of both systems of higher and reduced-order

While analysing the frequency response behaviour, it can be seen from figure 5.2 that the response of magnitude for both the systems is the same, and the phase response shows similar behaviour at low frequencies. Still, a slight variation is there in the phase of the reduced-order system at a higher frequency. But the GM's and phase PM's values depict that the variation in phase at high frequencies does not affect the value of GM and PM. The value of GM and PM is the same in both systems. So, from the comparison of the step and frequency response, it can be suggested that the reduced-order obtained in equation (5.3) is a good approximation of the higher-order system of equation (5.1).

The equivalency in the systems of the higher and reduced-order can is proved through obtaining the time moments of both systems. The equal value of the time moments of these two systems results that these systems are similar. The first three-time moments of both systems are obtained from equation (4.17) by putting $i=1, 2$ and 3. The first three-time moments of higher-order system are $T_{l1}= -1.08, T_{l2}=2.18$ and $T_{l3}= -6.39$ and first three-time moments of reduced-order system are $T_{r1}= -1.09,$ $T_{r2}= 2.27$ and $T_{r3}= -6.94$. It shows that the higher-order and reduced orders moments obtained by the implemented method are almost the same. Hence, it results that the 4th order system is similar to the obtained 2nd order system.

Comparative investigation: For checking the superiority of the implemented technique over the other literature techniques, a comparative investigation is performed. All techniques are used to reduce the given 4th order of equation (5.1) and a system of 2nd order. The step response is the base for comparison of the system of reduced-order, and it is obtained by numerous techniques, as depicted in figure 5.3. The figure shows that all techniques' transient response is almost the same, but Gautam's reduced-order system shows some variation in steady-state response. Furthermore, the comparison of step response characteristics as shown in figure 5.3 helps obtain the value of time-domain performance characteristics like settling time, rise time, steady-state error and overshoot, as highlighted in Table 5.2. The table shows that the value of rising time possessed by Tiwari, Narwal, Sikander (2015) and Parmar is similar to the implemented technique and original system. The most similar value of settling time exists for Tiwari, Sikander (2017), Narwal and Parmar. It is also equal to the original system with the system of reduced-order obtained by implemented technique. Moreover, the steady-state error's value is zero in all the systems except Narwal and Gautam. The value of overshoot is also zero for most of the systems except Sikander (2015) and Vishwakarma. So the combined study results that the newly developed technique and the technique presented by Tiwari, Sikander (2017) and Parmar shows better values of time-domain characteristics for step input which are equivalent to the higher-order system.

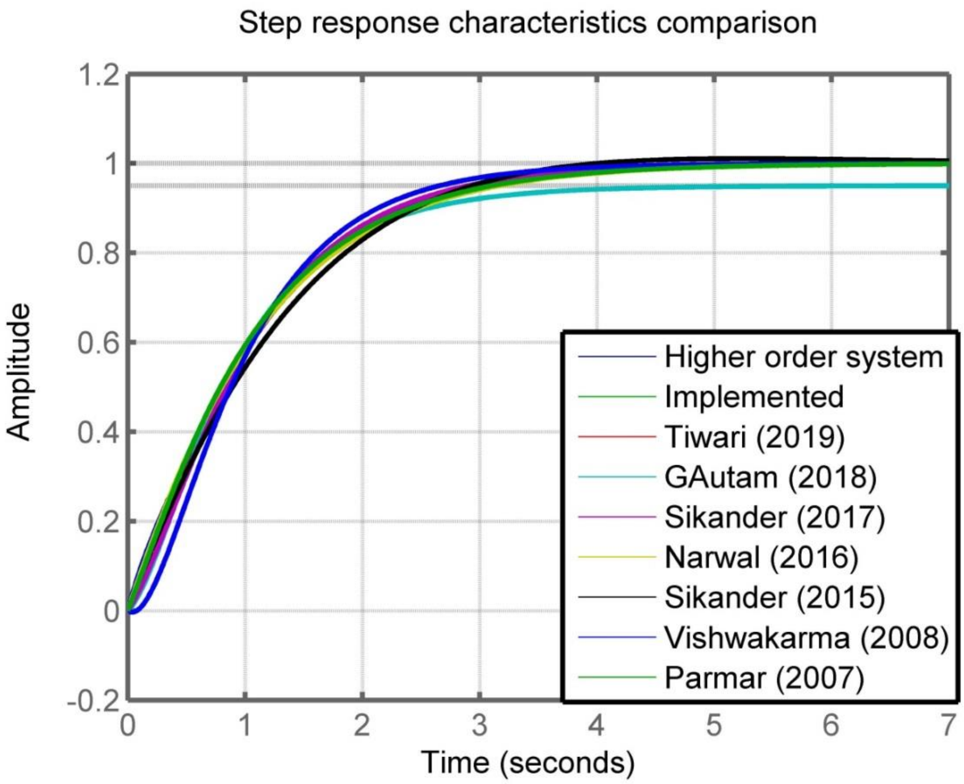


Figure 5.3. Comparative investigation of newly implemented technique with techniques given in the literature for the reduction of 4^{th} order single variable system based on step response characteristics

But, while obtaining the value of errors ISE, ITAE and IAE, it has resulted that the value of ISE is lower for the newly developed technique than all other techniques presented in the comparative study of Table 5.2. Also, the value of ITAE and IAE is observed as the minimum for the newly developed technique compared to other techniques used in the comparative study, as is depicted in the table. So, the comparison depicts that the developed method in the carried research shows a better value of time-domain performance characteristics than other techniques existing in literature, with the step input applied to the system of reduced-order.

From the reduction of system 4^{th} order to a 2^{nd} order system by the method implemented in the research work, it is clear that the method provides a good approximation of the system higher-order with similar time and frequency domain characteristics, equivalent time moments. Moreover, the obtained reduced-order approximation is also a stable system. Furthermore, the technique implemented here also provides better time-domain characteristics for step input than other techniques developed in the literature.

Table 5.2. Comparative exploration of errors and time-domain performance parameters of example 1 for a unit step input

Parameter	ISE $\times 10^{-04}$	ITAE	IAE	OS (%)	RT (sec.)	ST (sec.)	SSE
Actual	-	-	-	0	2.38	3.93	0
Implemented	1.815	0.058	0.027	0	2.32	4.06	0
Tiwari (2020)	2.60	0.0632	0.0281	0	2.32	4.07	0
Gautam (2019)	414.0	9.6873	0.8720	0	1.86	3.32	0.05
Sikander (2017)	14.0	0.0977	0.0581	0	2.09	3.70	0
Narwal (2017)	2.1335	0.0805	0.0307	0	2.33	4.08	0.0001
Sikander (2015)	28.0	0.4218	0.1319	1.07	2.30	3.41	0
Vishwakarma (2008)	80.0	0.2404	0.1455	0.32	1.83	3.34	0
Parmar (2007)	2.646	0.0403	0.0261	0	2.26	4.01	0

Example 2: Consider a 6^{th} order minimum phase SISO system of a hydropower system (Bansal et al., 2011; Vasu et al., 2020) having transfer function as shown in equation (5.4). This 6^{th} order system is to be reduced to the 3^{rd} order approximation.

$$G_6(s)=\frac{-0.1s^5-2.61s^4-7.3434s^3-4.8751s^2-0.0417s}{s^6+26.0999s^5+73.4331s^4+32.0833s^3+6.2497s^2+1.1666s+0.0417} \quad (5.4)$$

The first and the foremost step for obtaining the approximation of reduced-order is to expand using the partial fraction to get the poles and the MDIs of the given system. From expansion of equation (5.4), the poles of the system are $\sigma_1 = -22.9622$, $\sigma_2 = -2.6507$, $\sigma_3 = -0.3546$, $\sigma_4 = -0.044$, $\sigma_5 = -0.0442+0.2048i$ and $\sigma_6 = -0.0442-0.2048i$ and the MDI associated with each pole is $\eta_1 = 7.04\times10^{-06}$, $\eta_2 = 0.0059$, $\eta_3 = 0.3939$, $\eta_4 = 0.2040$ and $\eta_5 = \eta_6 = 0.0919$. Now, the 6th order system is to be converted into the 3rd order approximated system. As described in the methodology section, three clusters will be formed from six poles of the given system of higher-order. According to the values of MDI, the first cluster is formed of all real poles i.e. $\sigma_1 = -22.9622$, $\sigma_2 = -2.6507$, $\sigma_3 = -0.3546$, $\sigma_4 = -0.044$ and hence forming the cluster centre from equation (4.9) as $\sigma_{c1} = -0.0440$. The second cluster consists of only complex pole $\sigma_5 = -0.0442+0.2048i$ and similarly third cluster has only complex pole $\sigma_6 = -0.0442-0.2048i$ and the cluster centre of the second and third cluster are procured as $\sigma_{c2} = -0.0442+0.2048i$ and $\sigma_{c3} = -0.0442-0.2048i$ respectively. So, combining the three cluster centres and forming the 3rd order denominator equation of the system of reduced-order as described in equation (4.14).

$$D_3(s) = s^3 + 0.1324s^2 + 0.04779s + 0.001932$$

Now, the denominator equation of the system of the reduced-order is utilised for obtaining the numerator for the system of the reduced-order. The numerator is of 3rd order (A zero, s=0 is added in the numerator because of its presence in a higher-order system), having three unknown coefficients. So, the reduced-order system is displayed as follows:

$$G_3(s) = \frac{m_1 s^3 + m_2 s^2 + m_3 s}{s^3 + 0.1324s^2 + 0.04779s + 0.001932} \tag{5.5}$$

The next step is to find the value of unknown coefficients by genetic algorithm (GA). The integral square error in the step response for the system of higher-order equation (5.4) and the approximated system of reduced-order (5.5) is minimised by GA, and the value of unknown coefficients m_1, m_2 and m_3 are obtained. A total of 128 iterations are utilised to get the optimised results. After optimisation, the value of unknown coefficients is obtained as $m_1 = 0.0768$, $m_2 = -0.072$ and $m_3 = -0.00217$. By putting the value of the unknown coefficients, the equation for the reduced-order system is procured as follows:

$$G_3(s) = \frac{0.0768s^2 - 0.072s - 0.00217}{s^3 + 0.1324s^2 + 0.04779s + 0.001932} \tag{5.6}$$

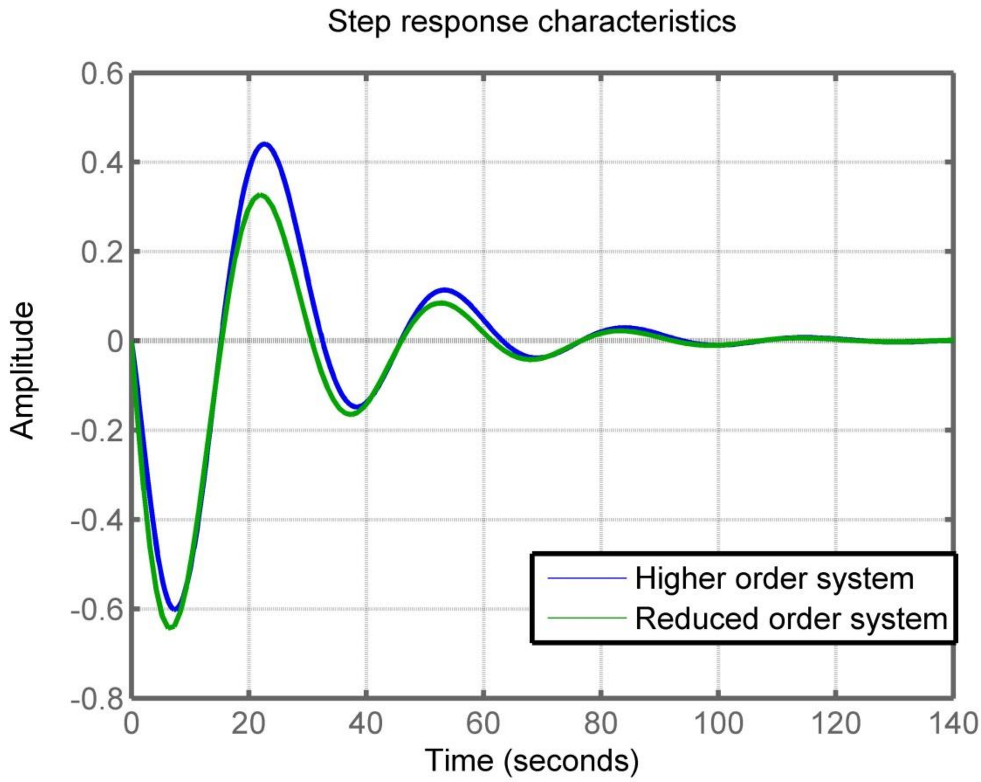


Figure 5.4. Characteristics of step response for the system of higher-order and reduced-order for example 2

The accuracy of the system of reduced-order obtained by equation (5.6) is checked by its comparison with the system of higher-order based on the frequency and step response behaviour of the system with higher and reduced order. The step response behaviour is depicted in figure 5.4, which shows that the higher-order system and reduced-order system shows the same behavioural characteristics. Initially, the response of both systems shows it undershoot of the approximately same amplitude and later on, both responses reach the final value. From Table 5.3, it is clear that the ISE, ITAE and IAE have low values, hence signifying that there is significantly less error between the systems of higher and lower orders. Also, the value of rising time, settling time, the first percentage undershoot, and steady-state error is noted for a unit step input. It is shown in Table 5.3, which shows that the values of rising time, the settling time for the higher and reduced-order system are almost the same. Moreover, the steady-state error in between both systems is zero with a higher value of percentage overshoot.

Table 5.3. Value of errors and various parameters to compare the system of 6th order system and obtained system of 3rd order for a unit step input

Parameter	ISE	ITAE	IAE	US (%)	RT (sec.)	ST (sec.)	SSE	GM	PM
HOS	-	-	-	∞	0	90.67	-	0.46	65.3
ROS	0.025	9.044	0.315	4.365×10^{18}	0.25	89.19	0	0.48	60.7

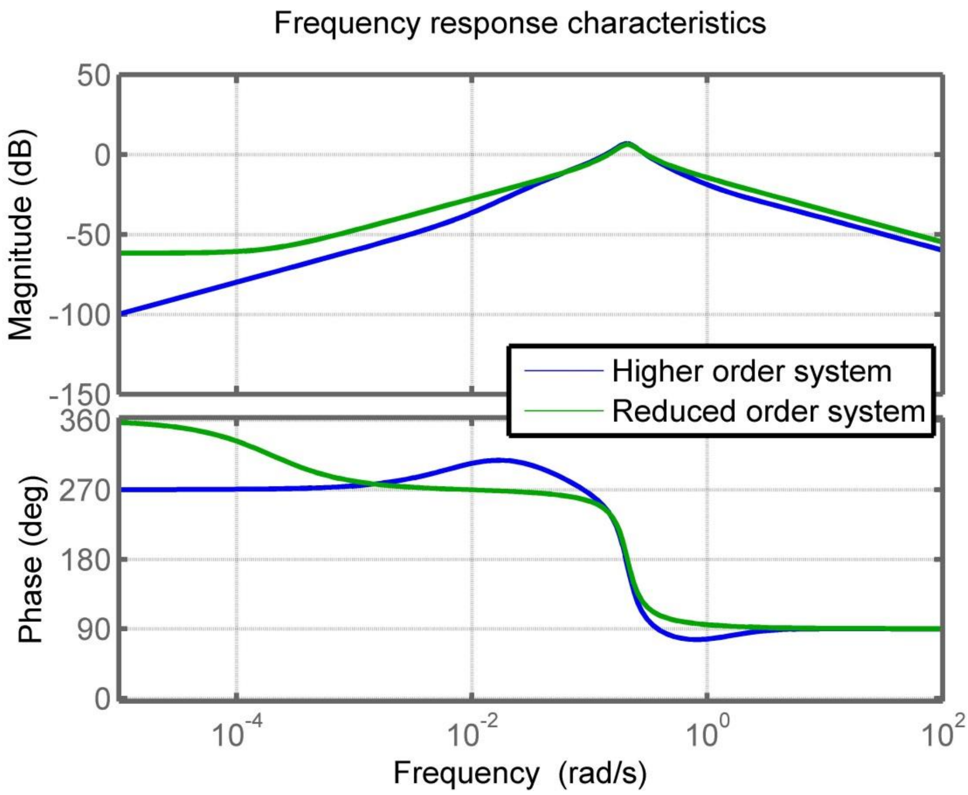


Figure 5.5. Example 2's frequency response characteristics for the system of higher-order system and system of reduced-order

While analysing the frequency response behaviour, it is seen from figure 5.5 that the response of magnitude for both the systems is the same, and the phase response shows similar behaviour at higher frequencies. Still, a slight variation is there in the phase of the reduced-order system at lower frequencies. But the GM and PM values in Table 5.3 show that the variation in phase at low frequencies does not affect the value of GM and PM. The value of GM and PM is almost the same in both systems. So, from the step and frequency response comparison, it can be suggested that the reduced-order obtained in equation (5.6) is an excellent 3rd order approximation of the higher-order system of equation (5.4).

The equivalency in the systems of higher and reduced-order is proved by obtaining the time moments of both systems. The equal value of the time moments of these two systems results that these systems are similar. The first three-time moments of both systems are obtained from equation (5.17) by putting *i=1, 2* and *3*. The first three-time moments of higher-order system are $T_{l1}= -1$, $T_{l2}= -177.86$ and $T_{l3}= 1477.05$ and first three-time moments of reduced-order system are $T_{r1}= -1.123$, $T_{r2}= -143.19$ and $T_{r3}= 1132.63$. It shows that the time moments of the system of higher and reduced-order obtained by implemented method is almost the same, and hence it results that the system of 6th order is similar to the obtained system of 3rd order.

Comparative investigation: For checking the superiority of the implemented technique over the other techniques studied in the literature, a comparative investigation is performed (Bansal et al., 2011; Narwal & Prasad, 2017; Sikander & Prasad, 2015, 2017; J. Singh et al., 2016; Vasu et al., 2012, 2020). All techniques are implied to reduce the given system of 6th order of equation (5.4) to a system of 3rd order. The comparison is conducted based on step response characteristics for s system of reduced-order obtained by vivid techniques, as depicted in figure 5.6. The figure shows that the transient response possessed by all techniques is almost the same. Still, the reduced-order system obtained by Sikander (2015) and Narwal shows higher variation in the transient response. Furthermore, the comparison of step response characteristics as shown in figure 5.6 hence helps to obtain time-domain performance characteristics like rising time, the value of first undershoot occurring in the response, settling time and steady-state error as highlighted in Table 5.4. The table shows that the value of rising time possessed by the technique implemented in the research work is 0.25, while for other techniques, its value is zero. Hence, the value of the first undershoot is decreased in the implemented technique. The techniques presented by Vasu and Bansal provide a considerable settling time which is not desirable. Moreover, the value of steady-state error is zero for all the techniques. So, the techniques presented in the research work and provided by Sikander, Singh and

Narwal are termed as beneficial for the comparative investigation based on performance characteristics of the time domain.

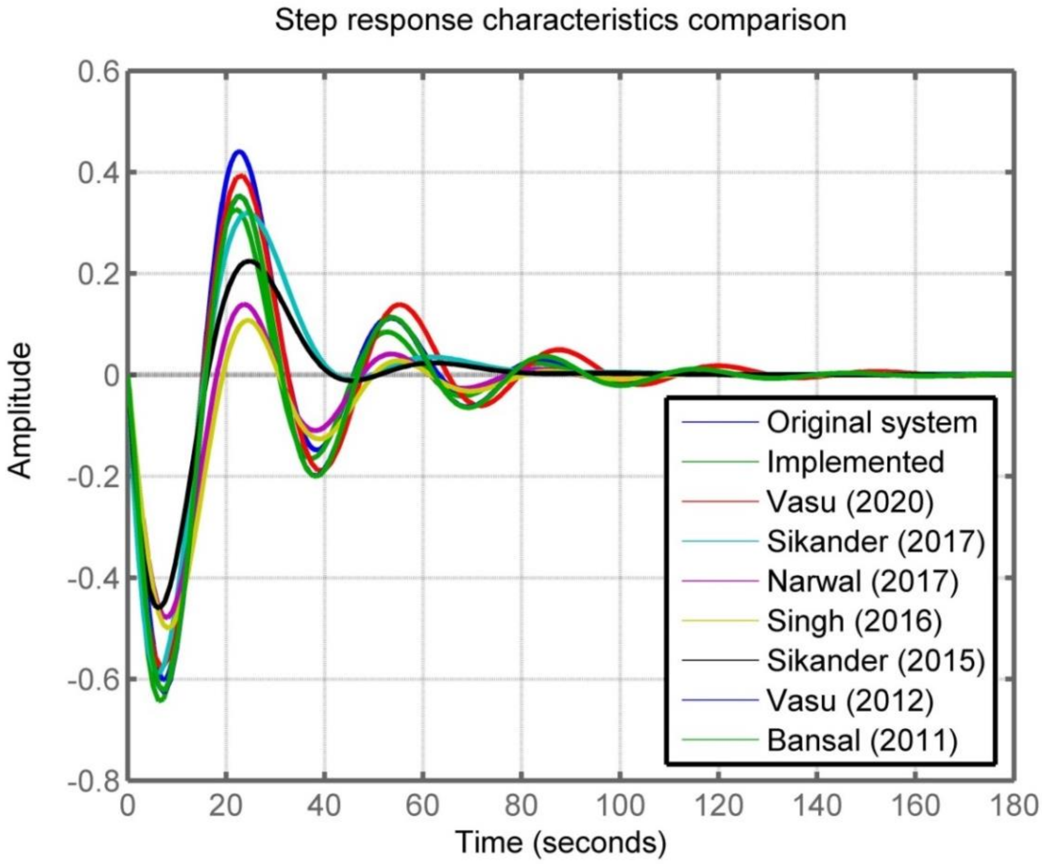


Figure 5.6. Comparative investigation of newly implemented technique with techniques given in the literature for the reduction of 6th order single variable system based on step response characteristics

While comparing the techniques based on the value of ISE, ITAE and IAE, it is visible from Table 5.4 that ISE for the implemented technique is lowest with the lowest value of ITAE and IAE errors. The technique of Narwal and Singh provides a significant value of all errors in comparison to the implemented system. Also, the reduced-order system's ISE's value obtained by Sikander (2017) is approximately 27 times higher than the implemented technique. The value of ISE for Sikander (2015) is 37 times higher than implemented technique. Hence, it results that the implemented technique provides very little error compared to other compared techniques. Based on the comparative investigation, the implemented technique is better than all techniques included when unit step input is applied to the system.

Table 5.4. Comparative exploration of errors and time-domain performance parameters, for example 2 for a unit step input

Parameter	ISE	ITAE	IAE	US (%)	RT (sec.)	ST (sec.)	SSE
Original	-	-	-	∞	0	90.67	-
Implemented	0.0252	9.0437	0.3152	4.365×10^{18}	0.25	89.19	0
Vasu (2020)	0.1044	18.558	0.3180	∞	0	125.1	0
Sikander (2017)	0.6949	26.348	0.6543	∞	0	76.06	0
Narwal (2017)	1.1361	19.557	0.6586	∞	0	86.11	0
Singh (2016)	1.4913	22.325	0.7423	∞	0	77.58	0
Sikander (2015)	0.9529	27.474	0.7487	∞	0	74.74	0
Vasu (2012)	0.1603	12.103	0.3158	∞	0	104.6	0
Bansal (2011)	0.1595	12.452	0.3175	∞	0	104.8	0

From the reduction of the system of the 6^{th} order system to the 3^{rd} order by the method implemented in the research work, it is clear that the method provides a good approximation of the system of the higher-order with similar time and frequency domain characteristics and equivalent time moments. Furthermore, the technique implemented here also provides better time-domain characteristics for step input than other techniques developed in the literature.

Example 3: A minimum phase SISO system of 6^{th} order (Prajapati & Prasad, 2020a) is considered for reducing by the methodology shown in the research work. The system's transfer function is shown as follows:

$$G_6(s) = \frac{2s^5 + 3s^4 + 16s^3 + 20s^2 + 8s + 1}{2s^6 + 33.6s^5 + 155.94s^4 + 209.46s^3 + 102.42s^2 + 18.3s + 1} \tag{5.7}$$

The first step of obtaining the reduced-order approximation is to perform partial fraction expansion to obtain poles and MDIs of the given system. From the expansion using partial fraction for equation (5.7), the six poles of the system are obtained as σ_1= *-0.1*, σ_2= *-0.2*, σ_3= *-0.5*, σ_4= *-1*, σ_5= *-5* and σ_6= *-10* and the MDI associated with each pole is η_1=*11.002*, η_2=*0.8435*, η_3=*0.0974*, η_4=*0.0772*, η_5= *0.0559* and η_6=*0.0222*. Now, the 6^{th} order system is to be converted into its 2^{nd} order approximated system. As described in the methodology section, two clusters will be formed from six poles of the given system of the higher-order. According to MDI values, the first cluster comprises one pole, i.e. σ_1= *-0.1*, forming the cluster centre as σ_{c1}= *−0.01*. The second cluster is formed from the remaining real poles as σ_2= *-0.2*, σ_3= *-0.5*, σ_4= *-1*, σ_5= *-5* and σ_6= *-10*. Hence the cluster centre of the second cluster is procured as σ_{c2}= *−0.2032* by solving the equation (4.9) for k=5. So, combining the two cluster centres and forming the denominator of 2^{nd} order, equation of system of reduced-order as described in equation (4.10).

$$D_2(s) = s^2 + 0.3032s + 0.02032$$

Now, the denominator equation of the system of reduced-order is utilised for obtaining the reduced-order system's numerator. Considering that the numerator is of 1^{st} order having two unknown coefficients. So, the system of reduced-order is displayed as follows:

$$G_2(s) = \frac{m_1 s + m_2}{s^2 + 0.3032s + 0.02032} \tag{5.8}$$

The next step is to find the value of unknown coefficients by genetic algorithm (GA). The ISE between the step response of the higher-order equation (5.7) system and the approximated system of reduced-order (5.8) is minimised by GA. Hence the value of unknown coefficients m_1 and m_2 are obtained. A total of 138 iterations are utilised to get the optimised results. After optimisation, the value of unknown coefficients is obtained as m_1= *0.0951* and m_2= *0.0203*. By putting the value of the unknown coefficients, the equation for the reduced-order system is procured as follows:

$$G_2(s) = \frac{0.0951s + 0.0203}{s^2 + 0.3032s + 0.02032} \tag{5.9}$$

The accuracy of the system of reduced-order obtained in equation (5.9) is checked by its comparison with the system of higher-order based on the frequency and step response behaviour of higher-order and reduced-order systems. Figure 5.7

shows the step response behaviour, which illustrates that, the system of higher-order and reduced-order shows the same behavioural characteristics in the all-time range. The unit step response of the system of reduced-order is precisely tracing the response of the system of the higher-order, as is clear from figure 5.7. Moreover, it is clear from Table 5.5 that the ISE, ITAE and IAE have shallow values, signifying very little error between the higher and lower-order systems. Furthermore, the value of settling and rise time for the system of higher-order and reduced-order is almost similar. Also, the overshoot exhibited by both systems is zero. Moreover, for unit step input, zero is the value of steady-state error. The step response comparison shows that the time domain characteristics of the reduced system 2^{nd} order perfectly match with the higher system of 6^{th} order from Table 5.5

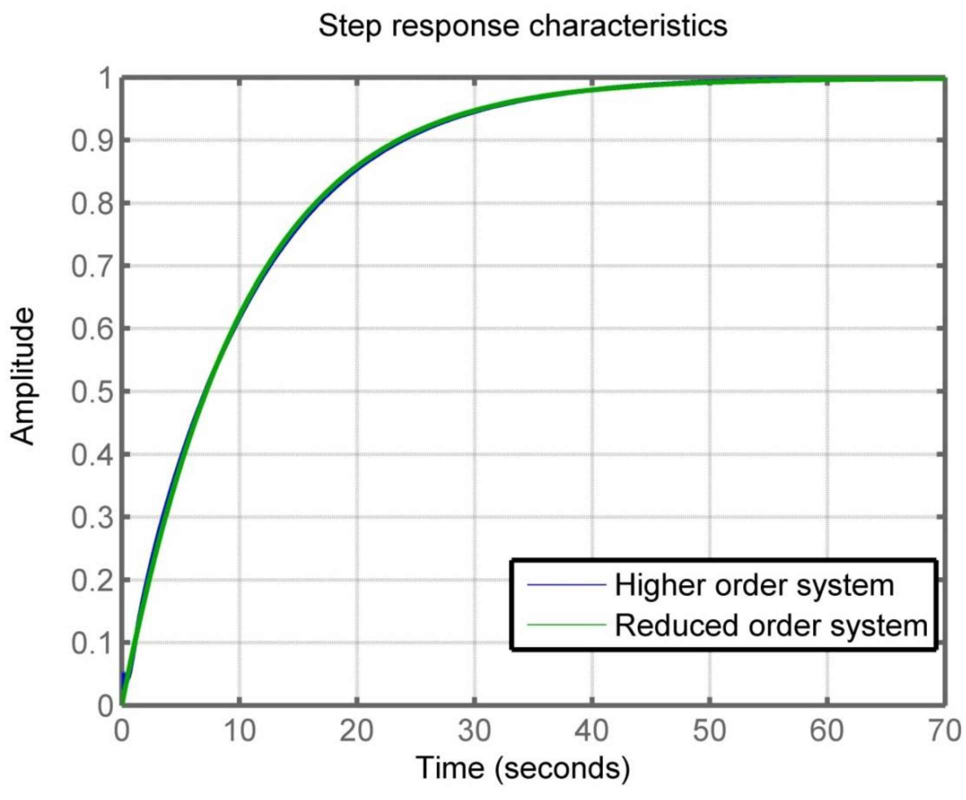


Figure 5.7. Step response characteristics of the system of higher-order and reduced-order for example 3

While analysing the frequency response behaviour, it is seen by figure 5.8 that the response of magnitude of both the systems is almost the same, and reduction removes spikes from the magnitude and shows a smooth magnitude response. The phase response of the system of reduced-order also shows a smooth behaviour by removing the sudden changes in phase at high frequency. But from Table 5.5, the value of GM is similar for the higher-order and system of reduced-order, and PM of the system of reduced-order is achieved much higher, thus removing any chances of instability in the system of reduced-order.

Table 5.5. Value of errors and various parameters to compare the higher system of 6th order and obtained system 3rd order for a unit step input

Parameter	ISE	ITAE	IAE	OS (%)	RT (sec.)	ST (sec.)	SSE	GM	PM
HOS	-	-	-	0	22.71	40.04	-	∞	180
ROS	1.10×10^{-03}	2.942	0.166	0	22.33	39.56	0	∞	∞

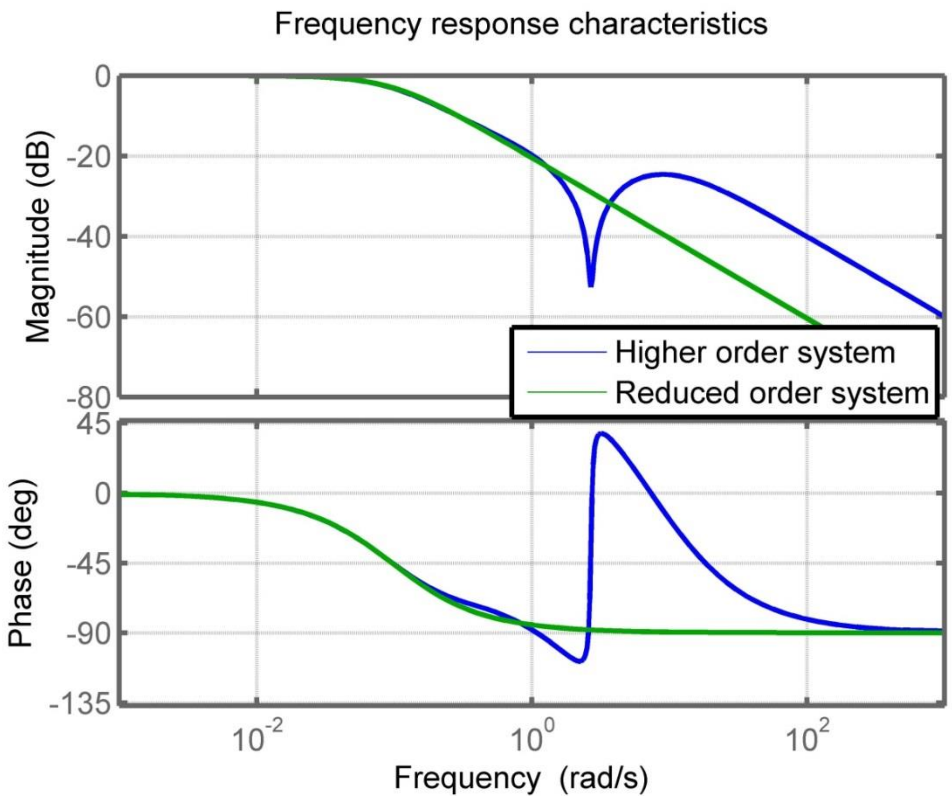


Figure 5.8. System of higher-order and reduced order's frequency response characteristics for example 3

So, it can be said through the frequency response characteristics that the higher and reduced-order systems are showing almost similar behaviour.

The equivalency in the systems of higher and reduced-order can be proved by obtaining the time moments of both systems. The similar value of the time moments of the two systems results that these systems are similar. The first three-time moments of both systems are obtained from equation (4.17) by putting *i=1, 2* and *3*. The first three-time moments of higher-order system are $T_{l1}= -10.3$, $T_{l2}= 212.14$ and $T_{l3}= -6477.7$ and first three-time moments of reduced-order system are $T_{r1}= -10.22$, $T_{r2}= 206.85$ and $T_{r3}= -6240$. It shows that the moments of time of the system of higher-order and reduced-order obtained by the implemented method is almost the same. Hence, it results that the 6^{th} order system is similar to the obtained 2^{nd} order system.

Comparative investigation: For checking the superiority of the implemented technique over the other techniques studied in the literature, a comparative investigation s is performed (Chongxin Huang et al., 2013; Prajapati & Prasad, 2020a; Sikander & Prasad, 2015a; N. Singh et al., 2006; Tiwari & Kaur, 2020a). All techniques are employed to reduce the given system of 6^{th} order of equation (5.7) and for obtaining a system of 2^{nd} order. The comparison is conducted based on step response characteristics of the system of reduced-order obtained through various techniques, as shown by figure 5.9. The figure depicts that the transient response possessed by all techniques is almost similar and tracing the similar path as traversed by the original system. Still, from the highlighted representation, it is clear that the original system shows a slight overshoot at the starting moment. And the response of all techniques removes this overshoot, except the technique designed by Tiwari. The response obtained by Tiwari is showing undershoot at the start of the response. The more precise comparative statement can be drawn from the time domain characteristics for step input and error between higher and reduced-order systems, as depicted in Table 5.6. The table shows that the rising time of the response exhibited by implemented technique and the Prajapati technique is almost similar to the rise time exhibited by the original system. Other techniques have variations in rising time. Also, while comparing the settling time, it is seen that the time to settle for the implemented and Prajapati technique is similar to the original system. Moreover, all the techniques other than Prajapati and implemented technique show some steady-state error which is not a permissible value.

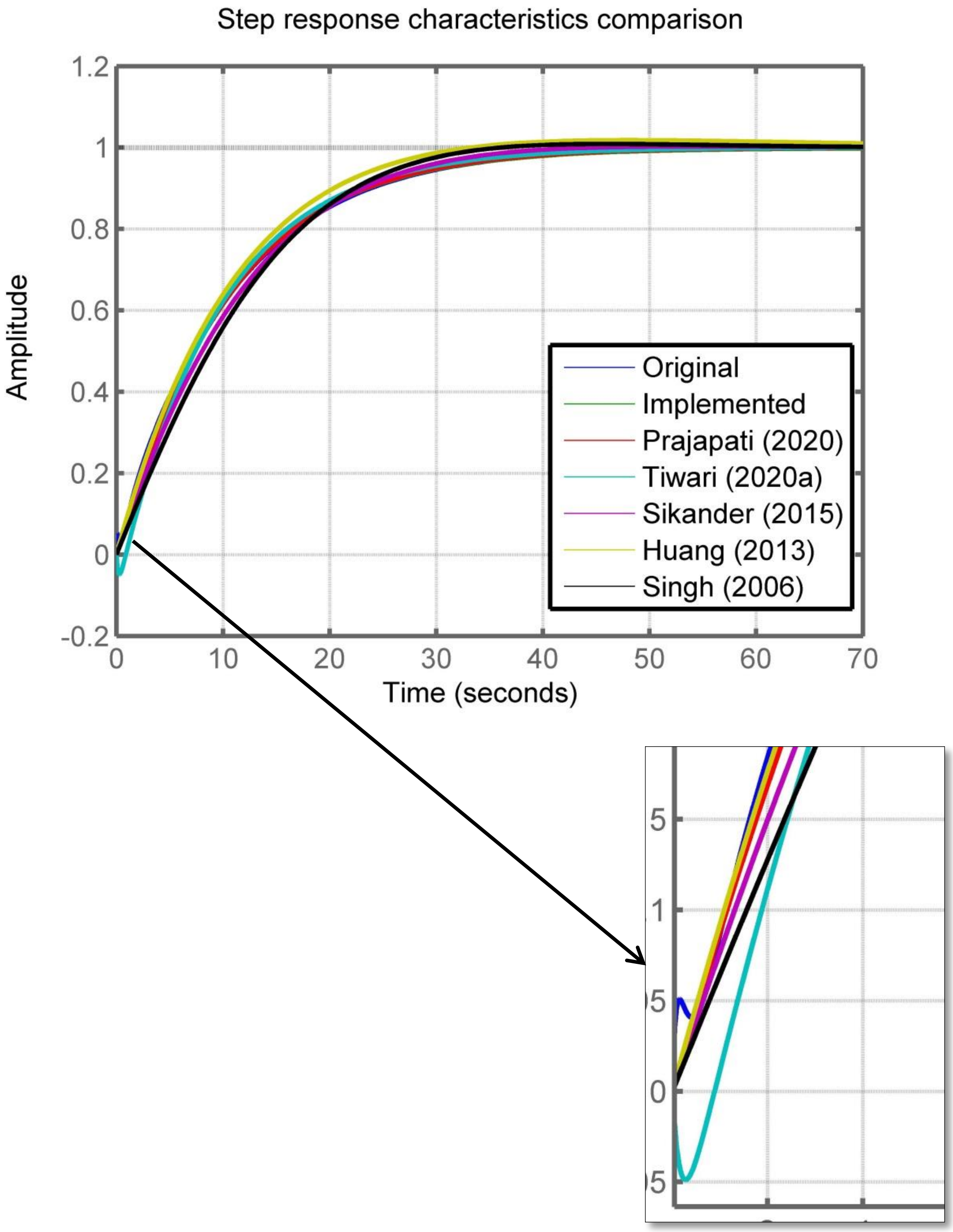


Figure 5.9. Comparative investigation of newly implemented technique with techniques given in the literature for the reduction of 6th order single variable system given in example 3 based on step response characteristics

Table 5.6. Comparative investigation of errors and time-domain performance parameters, for example 2 for a unit step input

Parameter	ISE	ITAE	IAE	OS (%)	RT (sec.)	ST (sec.)	SSE
Actual	-	-	-	0	22.71	40.04	-
Implemented	0.0011	2.94	0.16	0	22.33	39.56	0
Prajapati (2020)	0.0021	3.77	0.23	0	22.27	39.22	0
Tiwari (2020a)	0.0270	13.68	0.80	0	20.62	37.21	0.04
Sikander (2015)	0.0251	26.32	1.04	0.29	21.75	34.10	0.01
Huang (2013)	0.0609	77.22	2.04	1.79	19.29	28.75	0.23
Singh (2006)	0.0771	42.95	1.81	0.87	20.79	30.68	0.001

The techniques presented by Sikander, Huang and Singh also exhibit some overshoot that does not exist in the original system. Hence, it has resulted that the time domain performance characteristics of only implemented technique and the Prajapati technique are similar to the original higher-order system. While comparing the techniques based on the value of ISE, IAE and ITAE, then it is seen from Table 5.6, the ISE for the implemented technique is lower than other techniques used in the comparative investigation. Moreover, the reduced-order system obtained by Prajapati shows twice as ISE as the implemented technique. Hence, the implemented technique shows more accurate model order reduction with the least ISE than other techniques shown in the comparative investigation. Moreover, the value of ITAE and IAE errors is also lower as compared to other techniques. Hence, it results that the implemented technique provides very little error compared to other compared techniques. Based on the comparative investigation, the implemented technique is better than all techniques included when applied with the implemented technique unit step input.

From the reduction of the system of the 6th order system to the 2nd order by the method implemented in the research work, it is pretty straightforward; the method provides a good approximation of the higher-order system with similar time and frequency domain characteristics and equivalent time moments. Also, the implemented technique provides a stable reduced-order system. Furthermore, the technique implemented here also provides better time-domain characteristics for step input than other techniques developed in the literature.

Example 4: Consider an 8th order minimum phase SISO system having eight poles and seven zeros. This system is to be reduced to a system of 4th order system through the implemented technique. The transfer function of the higher-order system is given as follows:

$$G_8(s)=\frac{\begin{array}{c}19.82s^7+429.26156s^6+4843.8098s^5+45575.892s^4+241544.75s^3\\+905812.05s^2+1890443.1s+842597.95\end{array}}{\begin{array}{c}s^8+30.41s^7+358.4295s^6+2913.8638s^5+18110.567s^4+67556.983s^3\\+177383s^2+149172.19s+37752.826\end{array}} \tag{5.10}$$

The initial step for obtaining the reduced-order approximation is to perform partial fraction expansion of the higher-order system of equation (5.10) to obtain poles and MDIs of the given system. From partial fraction expansion of equation (5.10), the eight poles of the system are obtained as $\sigma_1 = -0.46$, $\sigma_2 = -0.75$, $\sigma_{3,4} = -2.2 \pm 3.6i$, $\sigma_{5,6} = -0.35 \pm 6.8i$, $\sigma_7 = -8.5$ and $\sigma_8 = -15.6$ and the MDI associated with each pole is $\eta_1 = 11.9$, $\eta_2 = 8.81$, $\eta_3 = 0.341$, $\eta_4 = 0.341$, $\eta_5 = 0.324$, $\eta_6 = 0.324$, $\eta_7 = 1.04$ and $\eta_8 = 0.576$. Now, the 8th order system is to be converted into its 4th order approximated system. As described in the methodology section, four clusters will be formed from eight poles of a given higher-order system. These four clusters are to be made so that two clusters contain complex poles and the other two clusters contain real poles. According to MDI values, the first cluster comprises one pole with the highest MDI, i.e. $\sigma_1 = -0.46$, forming the first cluster centre as $\sigma_{c1} = -0.46$. The formation of the second cluster is from the remaining real poles as $\sigma_2 = -0.75$, $\sigma_7 = -8.5$ and $\sigma_8 = -15.6$. Hence the cluster centre of the second cluster is procured as

$\sigma_{c2} = -0.759$ by solving the equation (4.9) for k=3 and most significant pole $\sigma_2 = -0.75$ based on the order of MDIs. The third and fourth clusters are formed by combining the complex poles lying in the same quadrant. Hence, from equations (4.11) and (4.12), the real parts and the imaginary parts of the complex cluster centres of tertiary and quaternary clusters are *-0.396* and ±*4.3i*, respectively. So, the cluster centres of tertiary and quaternary clusters are $\sigma_{c3} = -0.396+4.3i$, $\sigma_{c4} = -0.396-4.3i$. So, combining the first, second, third, and fourth cluster centres and results in the 4th order denominator equation of the reduced-order system is described in equation (4.14) for combining real and imaginary cluster centres. The equation for the denominator of the reduced-order is obtained as:

$$D_4(s) = s^4 + 2.01s^3 + 19.96s^2 + 23.01s + 6.511$$

Now, the denominator equation system of the reduced-order is utilised for obtaining the numerator of the system of reduced-order. Considering that the numerator is of 3rd order having four unknown coefficients. So, the system of reduced-order is displayed as follows:

$$G_4(s) = \frac{m_1 s^3 + m_2 s^2 + m_3 s + m_4}{s^4 + 2.01s^3 + 19.96s^2 + 23.01s + 6.511} \tag{5.11}$$

The next step is to find the value of unknown coefficients by genetic algorithm (GA). The integral square error between the higher order's step response (5.10) and the approximated system of reduced orders step response (5.11) is minimised by GA, and the value of unknown coefficients, i.e. m_1, m_2, m_3 and m_4 are obtained. A total of 159 iterations are utilised to get the optimised results. After optimisation, the value of unknown coefficients is obtained as $m_1 = 14.9$, $m_2 = 26.3$, $m_3 = 267$ and $m_4 = 145$. By putting the value of the unknown coefficients, the equation for the reduced-order system is procured as follows:

$$G_4(s) = \frac{14.9s^3 + 26.3s^2 + 267s + 145}{s^4 + 2.01s^3 + 19.96s^2 + 23.01s + 6.511} \tag{5.12}$$

The accuracy of the system of reduced-order found in equation (5.12) is checked by its comparison with the system of higher-order based on the frequency and step response behaviour of the system of higher-order and reduced-order. The step response behaviour is shown in figure 5.10, which shows that the system of higher-order and the system of reduced-order shows the same behavioural characteristics in the whole time range. The unit step response of the system of higher-order shows some oscillations before attaining the steady-state value. These oscillations are eliminated in a reduced-order system.

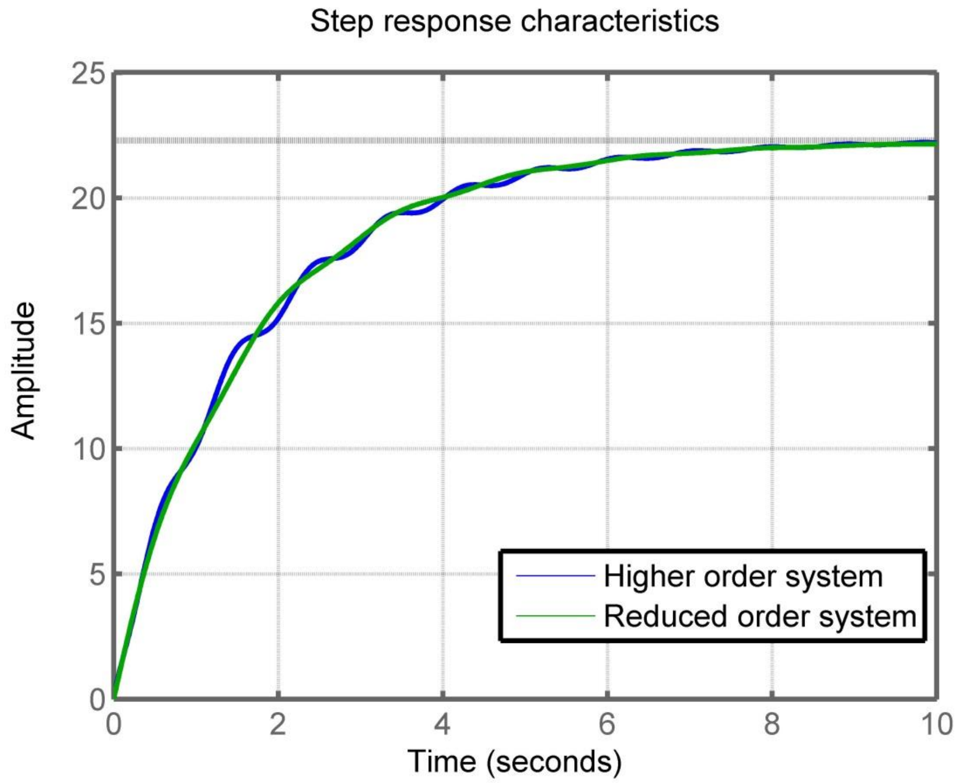


Figure 5.10. Step response characteristics of the system of higher-order system and the system of reduced-order for example 4

Moreover, it is clear from Table 5.7 that the ISE, ITAE & IAE has low value, signifying very little error between the higher and lower-order systems.

Table 5.7. Value of errors and various parameters to compare higher 8th order system and obtained 4th order system of example 4 for a unit step input

Parameter	ISE	ITAE	IAE	OS (%)	RT (sec.)	ST (sec.)	SSE	GM	PM
HOS	-	-	-	0	3.89	7.57	-	∞	29.77
ROS	0.513	4.307	1.442	0	3.87	7.22	0	∞	90.83

Furthermore, the system with higher-order and reduced-order has similar values for both rise and settling time. Also, the overshoot exhibited by both systems is zero. Moreover, for unit step input, the value of steady-state error is also noted as zero. The step response comparison shows that the time domain characteristics of the reduced system of 4^{th} order perfectly match with the higher system of 8^{th} order, shown in Table 5.7.

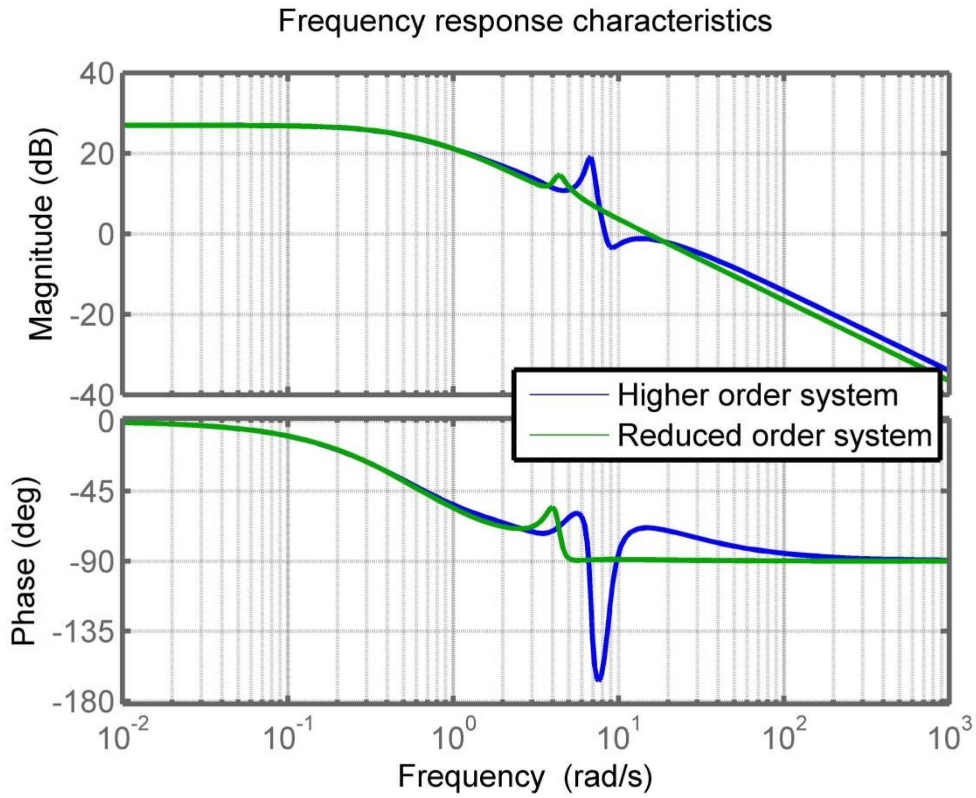


Figure 5.11. System of higher and reduced order's frequency response characteristics for example 4

While analysing the frequency response behaviour, it is seen from figure 5.11 that the magnitude of both the systems is almost the same, and reduction removes spikes from the magnitude and shows a smooth magnitude response. The system with reduced order's phase response also shows a smooth behaviour by removing the sudden changes in phase at high frequency. But from Table 5.7, the GM's value and PM's value are similar for both systems. So, from the frequency response characteristics, it can result that the higher and reduced-order systems are showing almost similar behaviour.

The equivalency in the system of higher and reduced-order is also proved by obtaining the moments for both systems. The similar value of the time moments of the two systems results that these systems are similar. Both systems' first three-time moments are obtained from equation (4.17) by putting the value of $i=1$, 2 and 3. The first three-time moments of the system of higher-order are $T_{l1}= -38.11$, $T_{l2}= 144.18$ and $T_{l3}= -860.10$, and three initial time moments of the system of reduced-order are $T_{r1}= -37.69$, $T_{r2}= 137.96$ and $T_{r3}= -796.91$. It shows that the time moments of the system of higher-order and reduced-order obtained by implemented technique is approximate. Hence, it results that the system of the 8th order system is similar to the derived system of the 4th order.

Comparative investigation: For checking the superiority of the implemented technique over the other techniques studied in the literature, a comparative investigation is performed (Glover, 1984; J. Singh et al., 2016; Tiwari & Kaur, 2016). All methods are used to reduce the given 8th order system of equation (5.10), and a 4th order reduced-order system is obtained. The comparison is conducted based on the characteristics of step response of the system of reduced-order obtained by numerous techniques, as depicted in figure 5.12. The figure shows that the transient response possessed by all techniques is almost similar and tracing the similar path as traversed by the original system. Still, from the highlighted representation, it is clear that the original system shows small oscillations in response before reaching its steady-state value, which is eliminated by a reduced-order system obtained by implemented technique. These oscillations are preserved by the Hankel norm approximation technique but show different steady-state values and provide steady-state error. The oscillations in response of Tiwari and Singh are higher than the original system.

The more precise comparative statement can be drawn from the value of time-domain characteristics for step input and error existing between higher and reduced-order systems, as depicted in Table 5.8. From the table, it is seen that the rising time of the response exhibited by implemented technique and all other techniques is almost similar to the rise time exhibited by the original system. Also, while comparing the settling time, it is observed that the time taken by the Hankel norm approximation technique to settle down is higher and deviated from the original response. Moreover, all the techniques other than implemented technique show some steady-state error which is not permissible for equivalency. The techniques presented by Tiwari also exhibit some overshoot that did not exist in the original system and the system of reduced-order obtained by all other techniques.

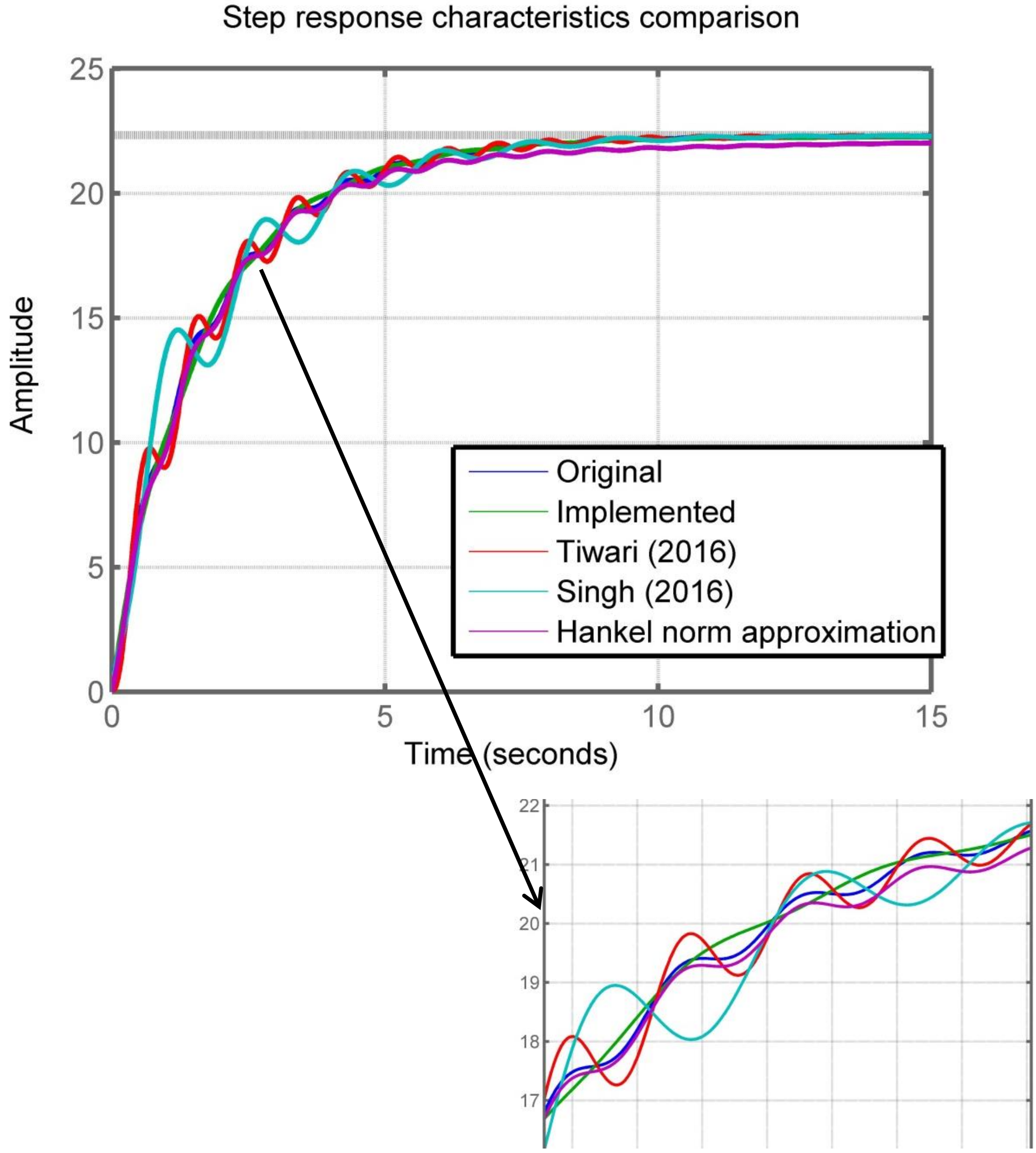


Figure 5.12. Basis step response characteristics comparative exploration of newly implemented technique with techniques given in the literature for the reduction of 8th order single variable system

While comparing the techniques based on the value of ISE, IAE and ITAE, it can be observed in Table 5.8 that ISE for the implemented technique is lower than other techniques used in the comparative investigation. The value of ISE is very much lower than that possessed by other techniques. Moreover, the value of ITAE and IAE errors shows a significant reduction for implemented technique. Hence, it results that the implemented technique provides significantly less error than other techniques. Based on the comparative investigation, the implemented technique can be termed better than all techniques included in the comparison, using unit step input to the system.

Table 5.8. Comparative exploration of errors and time-domain performance parameters, for example 4 for a unit step input

Parameter	ISE	ITAE	IAE	OS (%)	RT (sec.)	ST (sec.)	SSE
Actual	-	-	-	0	3.89	7.57	-
Implemented	0.513	4.307	1.442	0	3.87	7.22	0
Tiwari (2016)	1.766	8.345	2.829	0.002	3.84	7.68	0.003
Singh (2016)	8.558	16.899	5.736	0	3.87	7.33	0.002
Hankel norm	1.759	70.496	6.146	0	3.97	13.07	0.092

From the reduction of the system of the 8^{th} order to a 4^{th} order system by the technique implemented in the research work, it is visible that the method provides a relatively good approximation of the higher-order system with similar time and frequency domain characteristics and equivalent time moments. Also, the implemented technique provides a stable reduced-order system. Furthermore, the technique implemented here also provides better time-domain characteristics for step input than other techniques presented in the literature.

Example 5: Consider a single variable minimum phase system of 8^{th} order with the transfer function presented by equation (5.13). The 8^{th} order system is to be reduced to a system of 2^{nd} order by the technique implemented by the research work. The initial step for obtaining the reduced-order approximation is to perform partial fraction expansion of the higher-order system of equation (5.13) to obtain poles and MDIs of the given system.

$$G_8(s) = \frac{37s^7 + 1086s^6 + 13285s^5 + 82402s^4 + 278376s^3 + 511812s^2 + 482964s + 194480}{s^8 + 33s^7 + 437s^6 + 3017s^5 + 11870s^4 + 27470s^3 + 37492s^2 + 28880s + 9600} \quad (5.13)$$

By expanding the equation (5.13) by partial fraction, the system's eight poles are obtained as $\sigma_1 = -1$, $\sigma_{2,3} = -1 \pm j$, $\sigma_4 = -3$, $\sigma_5 = -4$, $\sigma_6 = -5$, $\sigma_7 = -8$ and $\sigma_8 = -10$ and the MDI associated with each pole is $\eta_1 = 10$, $\eta_2 = 4$, $\eta_3 = 4$, $\eta_4 = 3.41$, $\eta_5 = 9.877$, $\eta_6 = 8.51$, $\eta_7 = 8.35$ and $\eta_8 = 6.55$. Now, the 8th order system is to be converted into its 4th order approximated system. As described in the methodology section, four clusters will be formed from eight poles of a given higher-order system. These four clusters are to be made so that two clusters contain complex poles and two clusters contain remaining real poles. According to MDI values, the first cluster is formed of a single real pole with the largest MDI value, i.e. $\sigma_1 = -1$ and hence the cluster centre is obtained from a single-pole as $\sigma_{c1} = -1$. And remaining real poles $\sigma_4 = -3$, $\sigma_5 = -4$, $\sigma_6 = -5$, $\sigma_7 = -8$ and $\sigma_8 = -10$ form another cluster and forming the first cluster centre by the equation (4.9) for k=3 and most significant pole $\sigma_2 = -3$ as $\sigma_{c2} = -3.0164$. The tertiary and quaternary clusters are formed by the complex poles $\sigma_{2,3} = -1 \pm j$. So, the cluster centres of tertiary and quaternary clusters are $\sigma_{c3,4} = -1 \pm j$. So, combining the four cluster centres and forming the 4th order denominator equation of the reduced-order system as described in equation (4.14) for combining real and imaginary cluster centres. The denominator of the reduced-order is obtained as:

$$D_4(s) = s^4 + 6.016s^3 + 13.05s^2 + 14.07s + 6.033$$

Now, the denominator equation of the system of reduced-order is utilised for obtaining the system's numerator reduced-order. Considering that the numerator is of 3rd order having four unknown coefficients. So, the system of reduced-order is displayed as follows:

$$G_4(s) = \frac{m_1 s^3 + m_2 s^2 + m_3 s + m_4}{s^4 + 6.016s^3 + 13.05s^2 + 14.07s + 6.033} \quad (5.14)$$

The next step is to find the value of unknown coefficients by genetic algorithm (GA). The integral square error between the step response of the higher-order system equation (5.13) and the approximated reduced-order system (5.14) is minimised by GA, and the value of unknown coefficients, i.e. m_1, m_2, m_3 and m_4 are obtained. A total of 400 iterations are utilised to get the optimised results. After optimisation, the value of unknown coefficients is obtained as $m_1 = 33.8$, $m_2 = 152$, $m_3 = 221$ and $m_4 = 122.22$. By putting the value of the unknown coefficients, the equation for the reduced-order system is procured as follows:

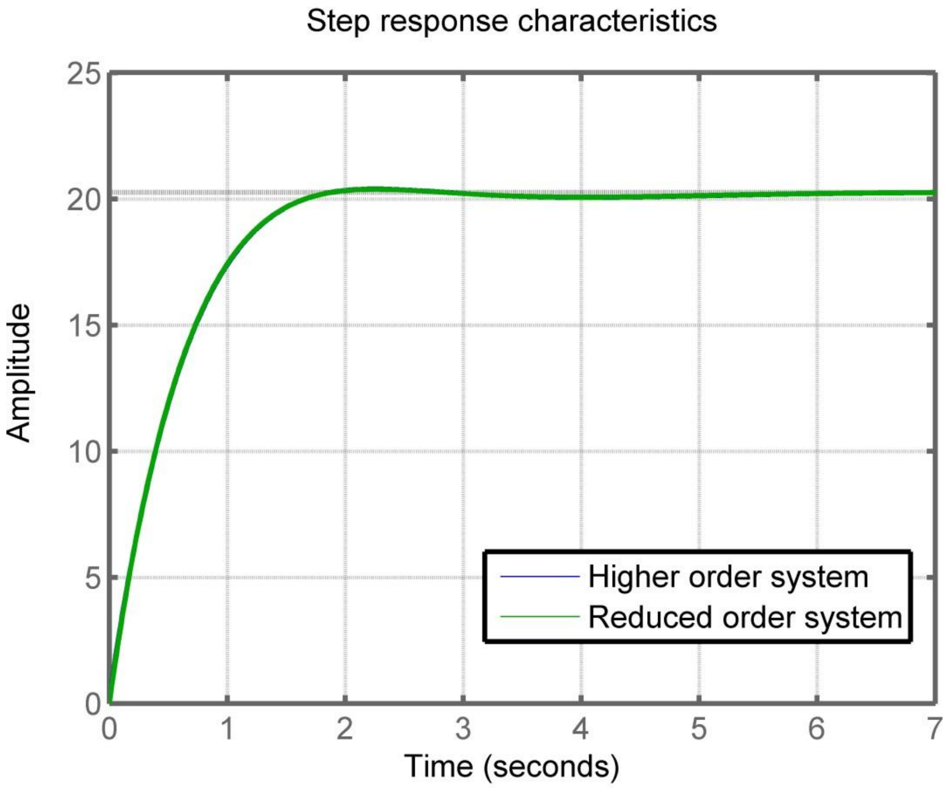


Figure 5.13. Step response characteristics of the higher-order system and reduced-order system, of example 5

$$G_4(s) = \frac{33.8s^3 + 152s^2 + 221s + 122.22}{s^4 + 6.016s^3 + 13.05s^2 + 14.07s + 6.033} \quad (5.15)$$

The accuracy of the system of reduced-order obtained in equation (5.15) is checked by its comparison with the system of higher-order based on the frequency and step response behaviour higher-order and reduced-order. Figure 5.13 shows the behaviour of the step response, which shows that the system of higher-order and reduced-order depicts the same behavioural characteristics in the whole time range. The response of the system of reduced-order overlaps with the higher order's. The more accurate conclusion on the step response behaviour can be drawn from Table

5.9. The table shows that the system of higher and reduced-order both have almost similar the value of rising time and settling time. Also, the overshoot exhibited by the system of reduced-order is less than that of the system of higher-order, but it is approximately similar in both systems. Moreover, the value of error at a steady state is zero when input is the unit step. The value of ISE, ITAE and IAE is low, as shown in Table 5.9, signifying very little error between the higher and lower-order systems. So, the step response comparison in figure 5.13 and Table 5.9 shows that the time-domain characteristics of the system of reduced 4^{th} order perfectly match with the system of higher 8^{th} order. Hence, the system of reduced-order is termed as a fair approximation of the system of higher-order.

Table 5.9. Value of errors and various parameters to compare higher 8^{th} order system and obtained 4^{th} order system of example 5 for a unit step input

Parameter	ISE	ITAE	IAE	OS (%)	RT (sec.)	ST (sec.)	SSE	G M	P M
HOS	-	-	-	0.64	1.07	1.58	-	∞	95.28
ROS	1.6×10^{-03}	0.105	0.061	0.59	1.06	1.58	0	∞	92.57

While analysing the frequency response behaviour, it can be seen from figure 5.14 that both the systems' phase and magnitude response is almost the same except for a slight deviation in phase response at higher frequencies. Moreover, the value of GM and PM from Table 5.9 depicts infinity and 95.28 respectively for the higher-order system and infinity and 92.57 for the reduced-order system. Hence the value of GM and PM is almost similar for the system of higher and reduced-order. So, from the frequency response characteristics, it can result that the higher and reduced-order systems are showing almost similar behaviour.

The equivalency in the system of higher and reduced-order can also be proved by obtaining both system's time moments. The similar value of the time moments of the two systems results that these systems are similar. Both three system's first three-time moments are obtained from equation (4.17) by putting the value of i=*1*, *2* and *3*.

Higher-order system's first three-time moments are $T_{l1}= -10.635$, $T_{l2}= 12.3807$ and $T_{l3}= -36.3548$ and system of reduced order's first three-time moments are $T_{r1}= -10.61$, $T_{r2}= 12.26$ and $T_{r3}= -35.58$. It shows that the time moments of the system of higher-order and reduced-order are obtained by implemented method is approximate. Hence, it results that the system of the 8th order is similar to the derived system of the 4th order.

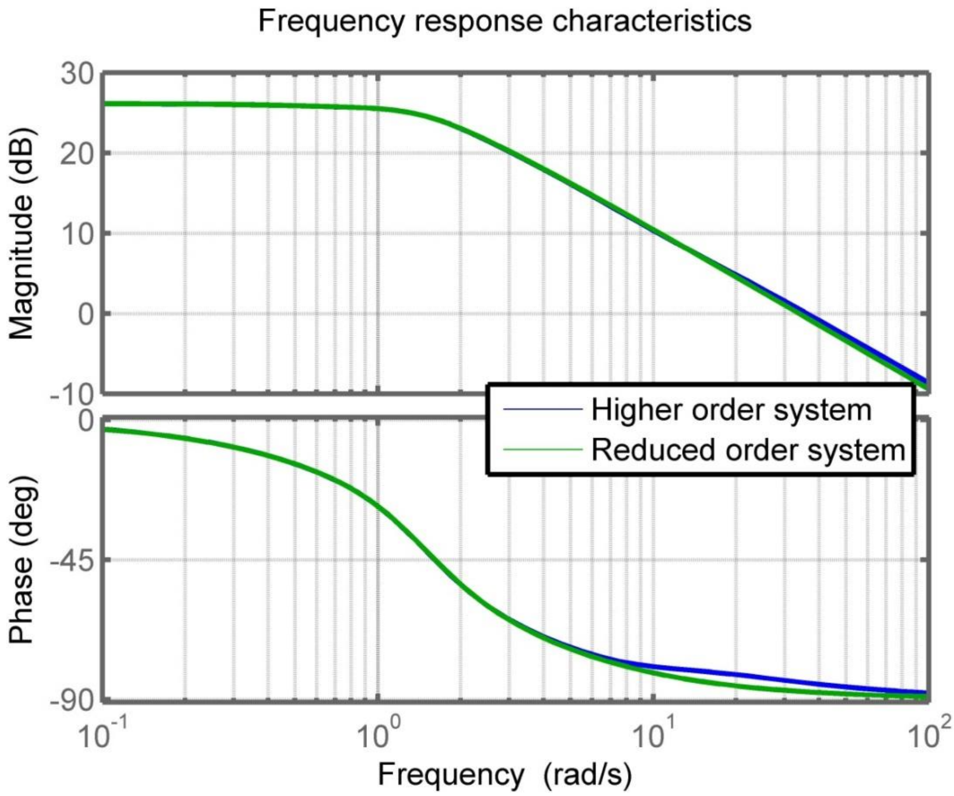


Figure 5.14. System of Higher and reduced order's frequency response characteristics for example 5

Comparative investigation: For checking the superiority of the implemented technique over the other techniques studied in the literature, a comparative investigation is performed (Prajapati & Prasad, 2020b; Sikander & Prasad, 2015a, 2017; J. Singh et al., 2016; Vishwakarma & Prasad, 2008) is performed to check the superiority of the implemented technique over other techniques. All methods are used to reduce the given 8th order system of equation (5.13), and a 4th order reduced-order system is obtained.

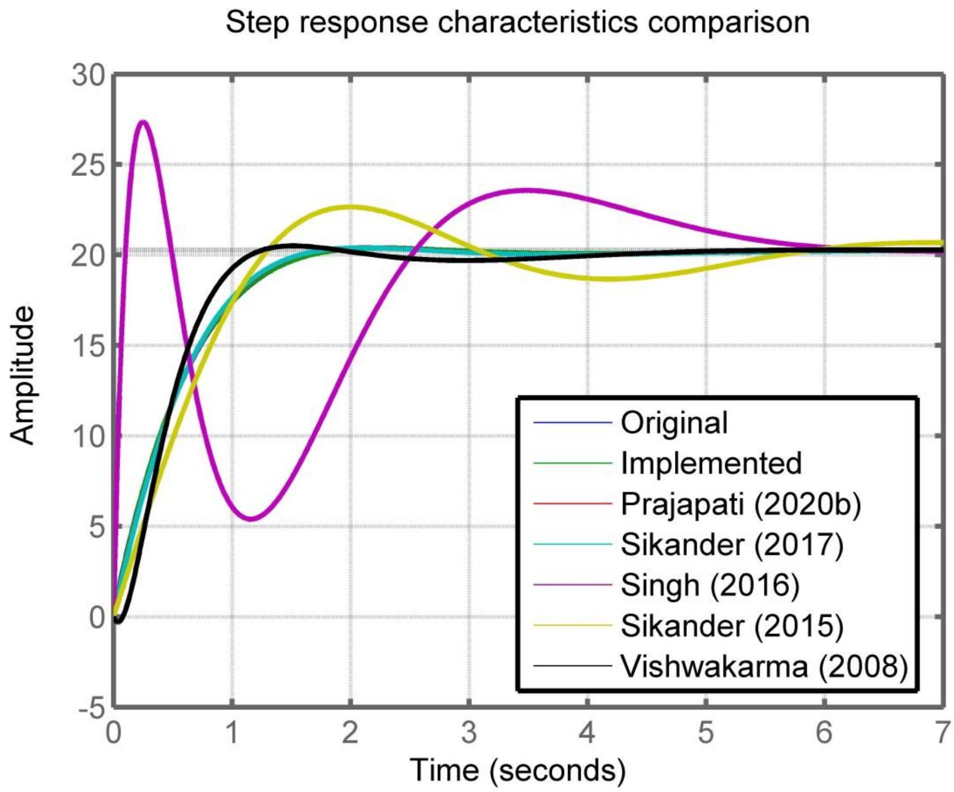


Figure 5.15. Comparative investigation based on step response characteristics of the newly implemented technique with techniques given in the literature for the reduction of 8th order single variable system of example 5

The step response characteristics are the basis for comparison of the system of reduced-order obtained by different techniques. Figure 5.15 show that the transient response possessed by implemented technique, Prajapati and Sikander (2017) shows the same behaviour as that possessed by the original system. Still, Sikander's (2015) and Vishwakarma response shows some oscillations before reaching their steady-state. Moreover, the response shown by Singh possesses higher oscillations which make this technique unsuitable. Also, it is visible in the figure that the steady-state of the Sikander (2015) system is not similar to the value provided by other techniques and the original system. This comparison signifies that the technique described by Prajapati and Sikander (2017) is only suitable for comparative measurement. The more precise comparative statement describes that the value of time-domain characteristics for step input and error existing between higher and reduced-order systems is depicted in Table 5.10. From the table, it is seen that the rise time of the response exhibited by the technique of Singh and Vishwakarma differs from the original system to a great extent. Hence, these techniques are not suitable to obtain the comparative statement.

Table 5.10. Comparative investigation of errors and time-domain performance parameters, for example 5 for a unit step input

Parameter	ISE	ITAE	IAE	OS (%)	RT (sec.)	ST (sec.)	SSE
Actual	-	-	-	0.64	1.07	1.58	-
Implemented	0.0016	0.1051	0.0618	0.59	1.06	1.58	0
Prajapati (2020b)	0.0583	0.7953	0.4253	0.68	1.03	1.51	0
Sikander (2017)	0.0881	0.9240	0.5153	0.66	1.02	1.50	0
Singh (2016)	299.79	50.662	29.858	34.98	0.07	5.60	0
Sikander (2015)	10.837	23.951	7.7324	11.80	0.97	7.12	0
Vishwakarma (2008)	4.8018	18.601	3.7157	1.18	0.70	3.69	0

Moreover, the implemented technique presented by Prajapati and Sikander (2017) shows the similarity in rising time with the higher-order system. The rise time exhibited by Sikander (2017) is almost similar to the original system, but the difference in settling time makes the technique unsuitable for the comparative investigation. The settling time exhibited by implemented technique and the technique of Prajapati and Sikander (2017) is almost similar to the original system with zero value of steady-state error. Hence, step response characteristics show that the implemented technique and the technique presented by Prajapati and Sikander (2017) are suitable for reducing the 8^{th} order system. While comparing the MOR techniques based on the value of ISE, IAE and ITAE, then from Table 5.10, the ISE for the implemented technique is 36 times lower than that possessed by the Prajapati technique and 55 times lower than that possessed by Sikander (2017). So, the value of ISE for implemented technique is much lower than that possessed by other techniques. Moreover, the value of ITAE and IAE errors shows a significant reduction for implemented technique. Hence, it results that the implemented technique provides significantly less error than other techniques. Based on the comparative investigation, the implemented technique is better than all techniques included when the input equal to the unit step is applied to the system.

From the reduction of the system of 8^{th} order to a system of 4^{th} order by the method used in the research work, it is clear that a good approximation is provided by the method of the system of higher-order with similar time and frequency domain characteristics equivalent time moments. Also, the implemented technique provides a stable reduced-order system. Furthermore, the technique implemented here also provides better time-domain characteristics for step input than other techniques presented in the literature.

Example 6: Applying the implemented technique of model order reduction to reduce a minimum phase 9^{th} order single variable system, and the given system is reduced in a 3^{rd} order approximation. The 9^{th} order system's transfer function is described as follows:

$$G_9(s)=\frac{s^4+35s^3+291s^2+1093s+1700}{s^9+9s^8+66s^7+294s^6+1029s^5+2541s^4+4684s^3+5856s^2+4620s+1700} \tag{5.16}$$

The initial step for obtaining the approximation reduced-order is to perform partial fraction expansion of the higher-order system of equation (5.16) to obtain pole and MDIs of the given system. From partial fraction expansion of equation (5.16), the nine poles of the system are obtained as $\sigma_1= -1$, $\sigma_{2,3}= -1\pm i$, $\sigma_{4,5}= -1\pm 2i$, $\sigma_{6,7}= -1\pm 3i$ and $\sigma_{8,9}= -1\pm i$ and the MDI is obtained for each pole as $\eta_1=1.5$, $\eta_2=0.0639$, $\eta_3=0.0639$, $\eta_4=0.2556$, $\eta_5=0.2556$, $\eta_6=0.0750$, $\eta_7=0.0750$, $\eta_8=0.0056$ and $\eta_9=0.0056$. Now, the 9^{th} order system is to be converted into its 3^{rd} order approximated system. As described in the methodology section, three clusters will be formed from nine poles of a given higher-order system. These three clusters are to be made so that two clusters contain complex poles and one cluster contains the remaining real pole. According to MDI values, the first cluster is formed of the available single real pole, i.e. $\sigma_1= -1$ and hence the cluster centre is obtained from a single-pole as $\sigma_{c1}= -1$. The second and third clusters are formed by placing all complex poles of the same quadrant in the same cluster and solving the equation (4.11) and (4.12) to get both clusters' real and imaginary parts. Hence, the cluster centres of second and third clusters are obtained as $\sigma_{c2,3} = -1 \pm 1.2262i$. So, combining the three cluster centres and forming the 3rd order denominator equation of the reduced-order system as described in equation (4.14) for combining real and imaginary cluster centres. The denominator equation of reduced-order can be obtained as:

$$D_3(s) = s^3 + 3s^2 + 4.504s + 2.504$$

Now, the denominator equation of the system of reduced-order is utilised for obtaining the system of reduced orders numerator. Considering that the numerator is of 2nd order having three unknown coefficients. So, the system of reduced-order is displayed as follows:

$$G_3(s) = \frac{m_1 s^2 + m_2 s + m_3}{s^3 + 3s^2 + 4.504s + 2.504} \qquad (5.17)$$

The next step is to find the value of unknown coefficients by genetic algorithm (GA). The ISE in step response of the system of the higher-order shown in equation (5.16) and the equation (5.17) that depicts the approximated reduced-order system is minimised by GA and thus obtaining the value of unrevealed coefficients, i.e. m_1, m_2 and m_3. A total of 78 iterations are utilised to get the optimised results. After optimisation, the value of unknown coefficients is obtained as $m_1 = -0.09615$, $m_2 = -0.9245$ and $m_3 = 2.504$. By putting the value of the unknown coefficients, the equation for the reduced-order system is procured as follows:

$$G_3(s) = \frac{-0.09615s^2 - 0.9245s + 2.504}{s^3 + 3s^2 + 4.504s + 2.504} \qquad (5.18)$$

The accuracy of the system of reduced-order obtained in equation (5.18) is checked by its comparison with the higher-order system of equation (5.16) based on the frequency and step response behaviour for the system of higher and reduced-order. The step response behaviour is depicted in figure 5.16, which shows that the higher-order and system of reduced-order display the same behavioural characteristics in the whole time range before reaching its value of steady-state. The response of the system of reduced-order shows little undershoot initially but then follows the response of the system of higher-order. Also, it is analysed from figure 5.16 that the response of the system of reduced-order utilises more time to enter the domain of steady-state as compared to the original system.

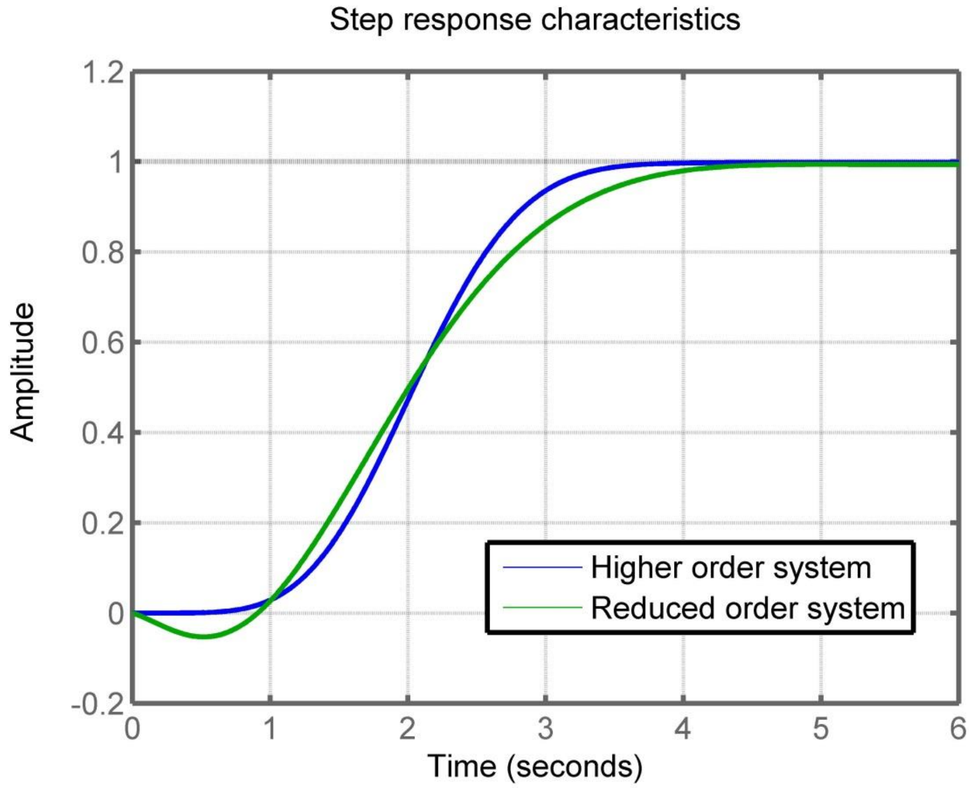


Figure 5.16. Step response characteristics of the system of the higher and reduced-order system, of example 6

The more accurate conclusion on the step response behaviour can be drawn from Table 5.11. It can be seen from the table, the value of rising time and settling time for the system of higher reduced-order is almost similar. Also, the overshoot exhibited by the system of reduced-order is zero, which is quite the same as the higher order's overshoot in the response. Moreover, the unit step input's steady-state error is also noted as zero in the system of reduced-order obtained by implemented technique. While analysing the error in the system of higher and reduced-order, it is observed that the value of ISE, ITAE and IAE is shallow. It signifies that there is significantly less error in between the system of higher and lower-order. So, the step response comparison in figure 5.16 and Table 5.11 shows the time-domain characteristics of the reduced system of 3^{rd} order approximately match with the higher system of 9^{th} order. Hence, the reduced-order system is termed as a relatively good approximation of the system of higher-order.

Table 5.11. Value of errors and various parameters to compare higher 9th order system and obtained 3rd order system of example 6 for a unit step input

Parameter	ISE	ITAE	IAE	OS (%)	RT (sec.)	ST (sec.)	SSE	GM	PM
HOS	-	-	-	0	1.53	3.35	-	1.52	180
ROS	9.4×10^{-03}	0.427	0.185	0	2.00	3.98	0	1.98	180

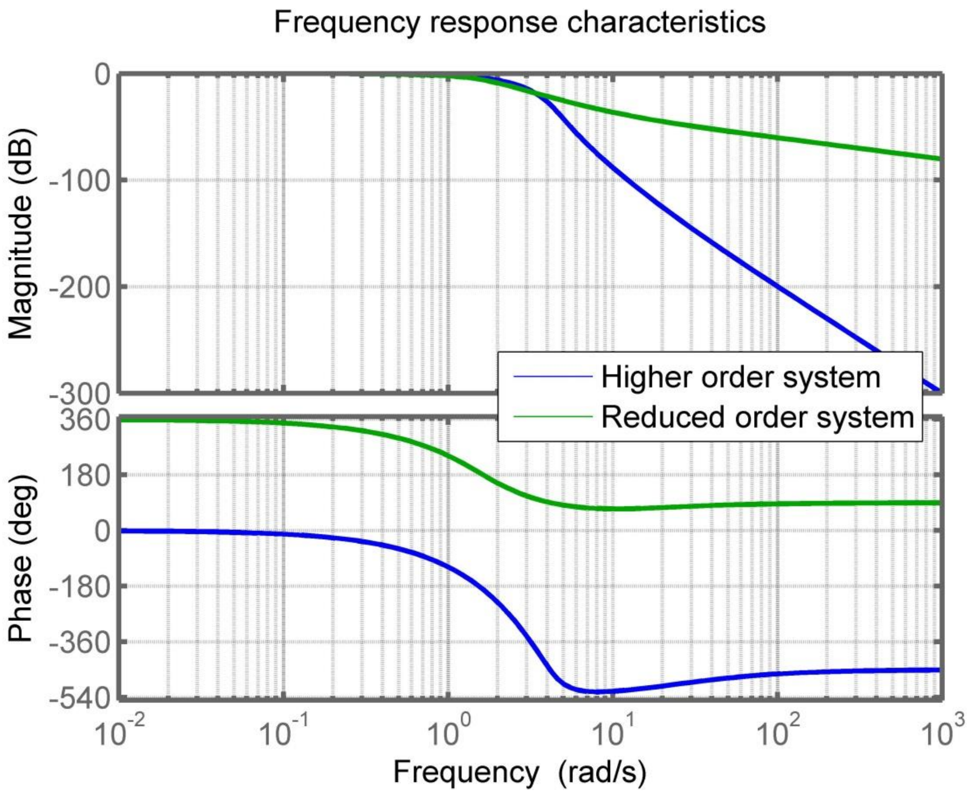


Figure 5.17. System of higher and reduced order's frequency response characteristics for example 6

While analysing the system of higher and reduced order's frequency response behaviour, it is seen from figure 5.17 that both systems' magnitude response is almost the same at low frequencies but shows a change in frequency at higher frequencies. Moreover, the phase response of the system of reduced-order has deviated with approximately 360^o phase shift. The main characteristic of frequency response, i.e., GM and PM, is noted from Table 5.11, which results that the value of GM and PM is almost similar for the higher and reduced-order system. So, from the frequency response characteristics, the frequency response of the system of higher-order varies after reduction, but the value of characteristics parameters does not change.

The equivalency in the system of higher and reduced-order is proved by obtaining the time moments of both systems. From the similar value of the two systems' time moments, these systems are similar. Both system's first three-time moments are obtained from equation (4.17) by putting the value of $i=1$, 2 and 3. Higher-order system's first three-time moments are $T_{l1}= -2.0747$, $T_{l2}= 4.8296$ and $T_{l3}= 13.0877$ and system of reduced order's first three-time moments are $T_{r1}= -2.16$, $T_{r2}= 5.1260$ and $T_{r3}= 14.5522$. It shows that the time moments of the system of higher-order and reduced-order obtained by implemented method is approximate. Hence, it results that the system of the 9th order is similar to the derived system of the 3rd order.

Comparative investigation: For checking the superiority of the implemented technique over the other techniques studied in the literature, a comparative investigation is performed (Glover 1984; Mukherjee, Satakshi, and Mittal 2005; Nasiri Soloklo, Hajmohammadi, and Farsangi 2015; Safonov and Chiang 1989; Tiwari and Kaur 2020a). All techniques are used to reduce the given system of 9th order as in equation (5.16), and a reduced system of 3rd order is obtained. The comparison is conducted based on the step response characteristics of the system of the reduced-order obtained by different techniques, as shown in figure 5.18. Figure 5.18 depicts that there is similar behaviour between the transient response characteristics of the Hankel norm technique and Schur technique as the original system of higher-order shows it. Still, the steady-state value differs from the original system by providing significant steady-state error's value. The step response of the Mukherjee technique shows the slow behaviour and hence taking a significant amount of time to reach its steady-state value. The response exhibited by the implemented technique and all other techniques used in comparison seems to be similar to that of the original higher-order of 9th order. It is clear from the highlighted portion of the step response characteristics that all reduction techniques show some overshoot or undershoot. The response of implemented technique and the technique of Tiwari shows only undershoot, while the response of Soloklo, Mukherjee and Schur shows some overshoot initially and then undershoot. The more precise comparative statement describes that the value of time-domain characteristics for step input and error existing between higher and reduced-order systems is depicted in Table 5.12.

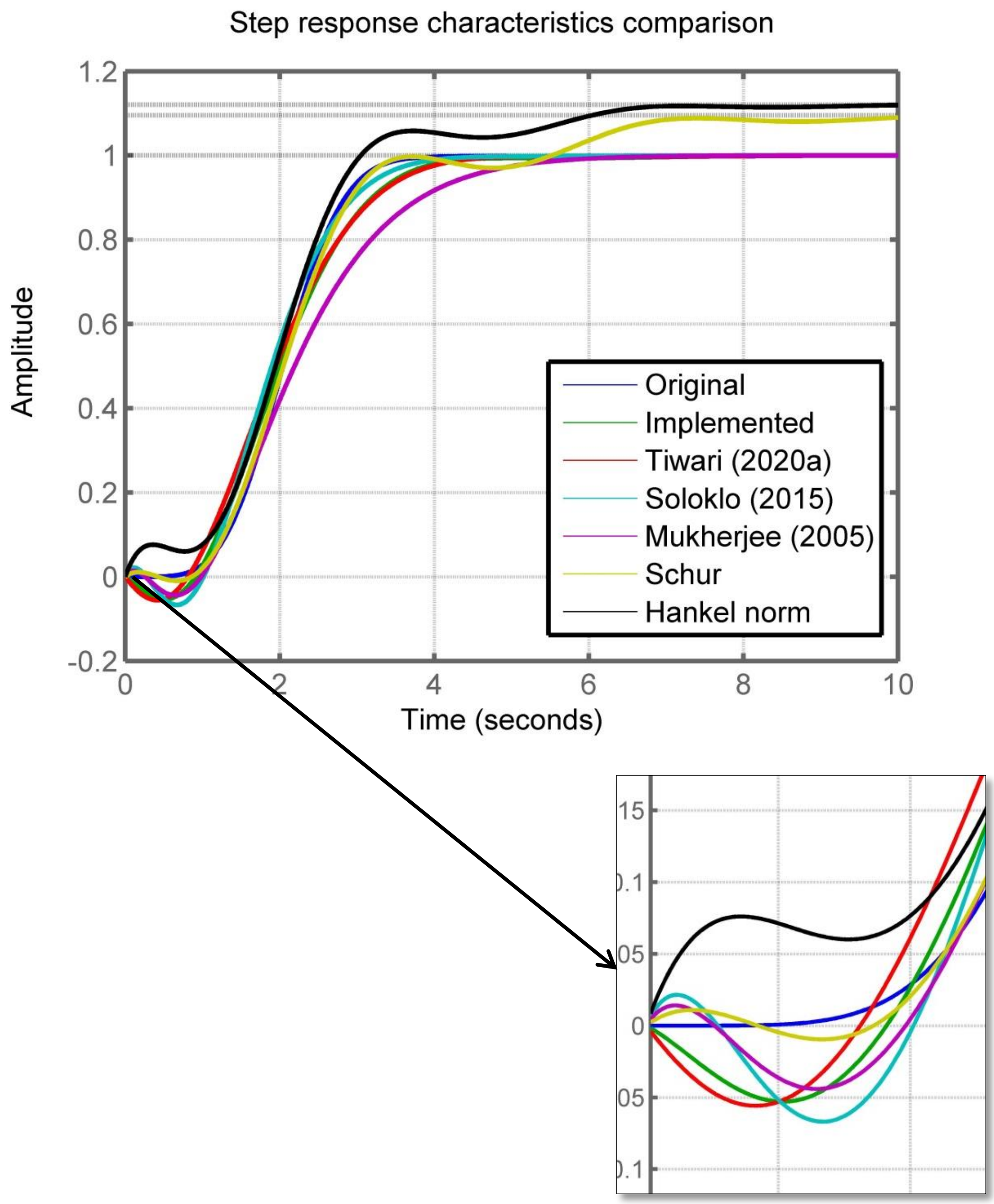


Figure 5.18. System of higher and reduced order's step response characteristics, of example 6

Table 5.12. Comparative exploration of errors and time-domain performance parameters, for example 6 for a unit step input

Parameter	ISE $\times 10^{-05}$	ITAE $\times 10^{-03}$	IAE $\times 10^{-03}$	OS (%)	RT (sec.)	ST (sec.)	SSE
Original	-	-	-	0	1.53	3.35	-
Implemented	0.0094	0.427	0.185	0	2.00	3.98	0
Tiwari (2020a)	0.0148	0.4906	0.2260	0	2.12	4.05	0
Soloklo (2015)	0.0098	0.3131	0.1645	0	1.72	3.68	0
Mukherjee (2005)	0.0427	1.1488	0.3750	0	2.54	5.17	0
Schur Technique	0.0283	2.8951	0.4043	0.0072	2.10	6.62	0.09
Hankel norm	0.0753	4.8031	0.8154	0	1.92	6.10	0.12

From Table 5.12, it can be seen that the rise time of the response exhibited by the technique of Mukherjee is higher than the original system to a great extent. Hence, this technique is not suitable to obtain the comparative statement. Moreover, the implemented technique and that presented by Soloklo and Tiwari show the similarity in rising time with the higher-order system. The rise time exhibited by Schur and Hankel norm approximation is approximately similar to the original system. Still, the value of settling time and steady-state error makes these techniques less attractive than other techniques. The settling time exhibited by implemented technique and Tiwari and Soloklo is almost similar to the original system with zero value of steady-state error. The MOR techniques are compared based on the ITAE, ISE and IAE, and Table 5.10 shows that the ISE for the implemented technique is almost similar to the ISE exhibited by Soloklo. In contrast, other techniques show a higher value of ISE. Similar results are obtained for the other errors ITAE and IAE. Hence, the implemented technique and the technique of Soloklo provide significantly less error than other techniques. Based on the comparative investigation, these two techniques can be termed better than all techniques included in the comparison when the system is provided with a unit step input.

From the reduction of the system of 9th order to obtain a system of 3rd order by the method implemented in the research work, it is clear that the method implemented gives an approximation that is similar for the system of higher-order with similar time and frequency domain characteristics and equivalent time moments. Also, the implemented technique provides a stable reduced-order system. Furthermore, the technique implemented here also provides better time-domain characteristics for step input than other techniques presented in the literature.

Example 7: Consider a minimum phase SISO system of 10th order which is to be reduced to a system of 3rd order by the technique presented in the research. The system of 10th order's transfer function is represented as:

$$G_{10}(s)=\frac{\begin{array}{c}(s+7.25)(s+16.66)(s+28.47)(s+42.47)(s+58.24)(s+75.68)\\(s+94.55)(s+114.71)(s+136.02)\end{array}}{\begin{array}{c}(s+3.11)(s+10.68)(s+20.68)(s+32.99)(s+47.32)(s+63.42)\\(s+81.05)(s+100.07)(s+120.31)(s+141.66)\end{array}} \quad (5.19)$$

The initial step for obtaining the approximation of reduced-order is to perform partial fraction expansion of the higher-order system of equation (5.19) to obtain poles and MDIs of the given system. From partial fraction expansion of equation (5.19), the ten poles of the system are obtained as $\sigma_1= -3.11$, $\sigma_2= -10.68$, $\sigma_3= -20.68$, $\sigma_4= -32.99$, $\sigma_5= -47.32$, $\sigma_6= -63.42$, $\sigma_7= -81.05$, $\sigma_8= -100.07$, $\sigma_9= -120.31$ and $\sigma_{10}= -141.66$ and the MDI is obtained for each pole as $\eta_1=0.075$, $\eta_2=0.0124$, $\eta_3=0.0055$, $\eta_4=0.0027$, $\eta_5=0.0017$, $\eta_6=0.0011$, $\eta_7=0.00085$, $\eta_8=0.00066$, $\eta_9=0.00054$ and $\eta_{10}=0.00053$. Now, the 10th order system is to be converted into its 3rd order approximated system. As described in the methodology section, three clusters will be formed from ten poles of a given higher-order system. These three clusters are to be made so that each cluster contains a set of real poles. According to MDI, the first cluster is formed of a single real pole, i.e. $\sigma_1= -3.11$ and hence the cluster centre of the first cluster is obtained as $\sigma_{c1}= -3.11$. The second cluster comprises another real pole $\sigma_2= -10.68$ and forming the cluster centre as $\sigma_{c2}= -10.68$. The value of cluster centre of third clusters is obtained by placing all remaining poles i.e. $\sigma_3= -20.68$, $\sigma_4= -32.99$,

σ_5= *-47.32,* σ_6= *-63.42,* σ_7= *-81.05,* σ_8= *-100.07,* σ_9= *-120.31* and σ_{10}= *-141.66* in the third cluster and solving the equation (4.9). Hence, the cluster centre of third clusters is obtained as σ_{c3}= *-20.68.* So, combining the three cluster centres and forming the 3rd order denominator equation of the system of reduced-order as described in equation (4.10). The denominator equation of the reduced-order is obtained as:

$$D_3(s) = s^3 + 34.47s^2 + 318.4s + 686.9$$

Now, the denominator equation of the system of reduced-order is utilised for obtaining the numerator of the system of reduced-order. Considering that the numerator is of 2nd order having three unknown coefficients. So, the system of reduced-order is displayed as follows:

$$G_3(s) = \frac{m_1 s^2 + m_2 s + m_3}{s^3 + 34.47s^2 + 318.4s + 686.9} \tag{5.20}$$

The next step is to find the value of unknown coefficients by genetic algorithm (GA). The integral square error in the system of higher order's step response equation (5.19) and the approximated system of reduced-order (5.20) is minimised by GA, and the value of unknown coefficients, namely m_1, m_2 and m_3 are determined. A total of 62 iterations are utilised to get the optimised results. After optimisation, the value of unknown coefficients is obtained as m_1= *0.0994*, m_2= *14.8*, and m_3= *69.4*. By putting the value of the unknown coefficients, the equation for the reduced-order system is procured as follows:

$$G_3(s) = \frac{0.0994s^2 + 14.8s + 69.4}{s^3 + 34.47s^2 + 318.4s + 686.9} \tag{5.21}$$

The system of reduced order's accuracy, which is obtained in equation (5.21), is checked by its comparison with the higher-order system of equation (5.19) based on the frequency and step response behaviour of the system of higher-order and reduced-order. The behaviour of step response is depicted in figure 5.19, which shows that the system of higher-order and reduced-order depicts the same behavioural characteristics in the whole time range. Moreover, the steady-state value of both systems is seen to be nearly equal to the step response behaviour. The more accurate conclusion on the step response behaviour can be drawn from Table 5.13. It is visible that rise time and settling time's value for the system of higher and reduced-order is almost similar. Also, the overshoot exhibited by the system of reduced-order is zero, which is similar to the response's overshoot of the system of higher-order. Moreover, for the unit step input, steady-state error's value is also noted as zero in the system of reduced-order obtained by implemented technique. While analysing the error in the system of higher and reduced-order, it is seen that the value of ISE, ITAE & IAE is shallow. It signifies that there is significantly less error in the system of higher and lower-order. So, from figure 5.19 and Table 5.13, it is seen that the time-domain characteristics of the reduced system of 3rd order approximately matches with the higher system of 10th order. Hence, the system of reduced-order is termed as a reasonably good approximation of the system of higher-order.

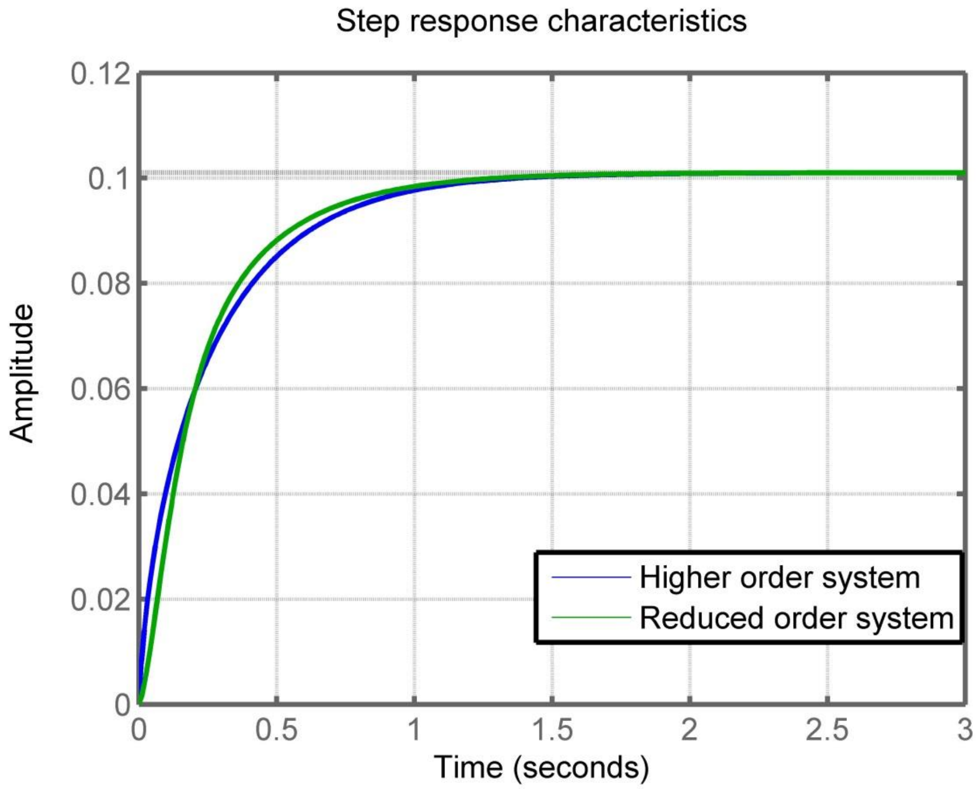


Figure 5.19. System of higher and reduced order's step response characteristics, for example 7

While analysing the system of higher and reduced order's frequency response behaviour from figure 5.20, it is revealed that both the system's magnitude response is almost the same at low frequencies but showing a change in magnitude at higher frequencies. Moreover, the system of reduced order's phase response has deviated from the system of higher-order for a particular band of frequency. The main characteristic of frequency response, i.e., GM and PM, is noted from Table 5.13, which results that the value of GM and PM is similar for the higher and reduced-order system. So, the frequency response characteristics depict that the system of higher order's frequency response varies after reduction, but the value of characteristics parameters does not change.

Table 5.13. Value of errors and various parameters to compare higher 10th order system and obtained 3rd order system of example 7 for a unit step input

Parameter	ISE	ITAE	IAE	OS (%)	RT (sec.)	ST (sec.)	SSE	GM	PM
HOS	-	-	-	0	0.63	1.16	-	∞	∞
ROS	1.92×10^{-05}	1.4×10^{-03}	3.5×10^{-03}	0	0.53	1.08	0	∞	∞

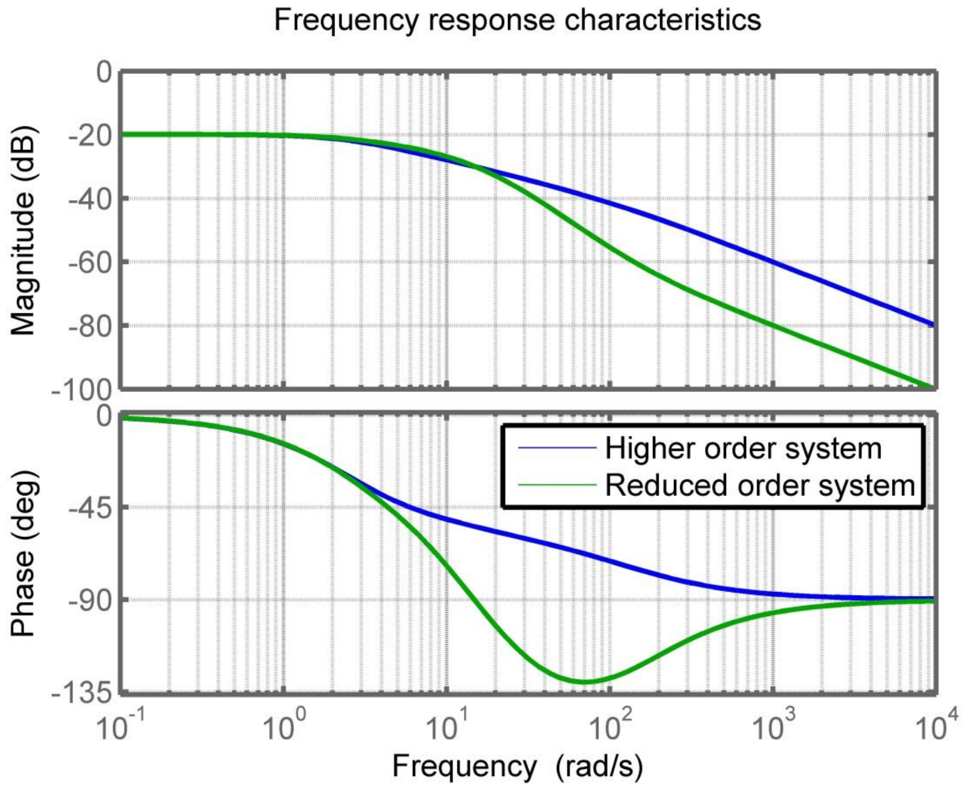


Figure 5.20. System of higher and reduced order's frequency response characteristics, for example 7

The equivalency in the system of higher and reduced-order can be proven by obtaining both the systems' time moments. The similar value of the time moments of the two systems results that these systems are similar. For both systems' the initial three-time moments are obtained from equation (4.17) by putting the value of *i=1, 2* and *3*. The system of higher order's first three-time moments are $T_{l1}= -0.0257$, $T_{l2}= 0.0158$ and $T_{l3}= -0.0151$ and the system of reduced order's first three-time moments are $T_{r1}= -0.0253$, $T_{r2}= 0.0136$ and $T_{r3}= -0.0122$. It shows that the time moments of the system of higher-order and reduced-order obtained by the applied method is approximate. Hence, it results that the system of 10th order is similar to the derived system of 3rd order.

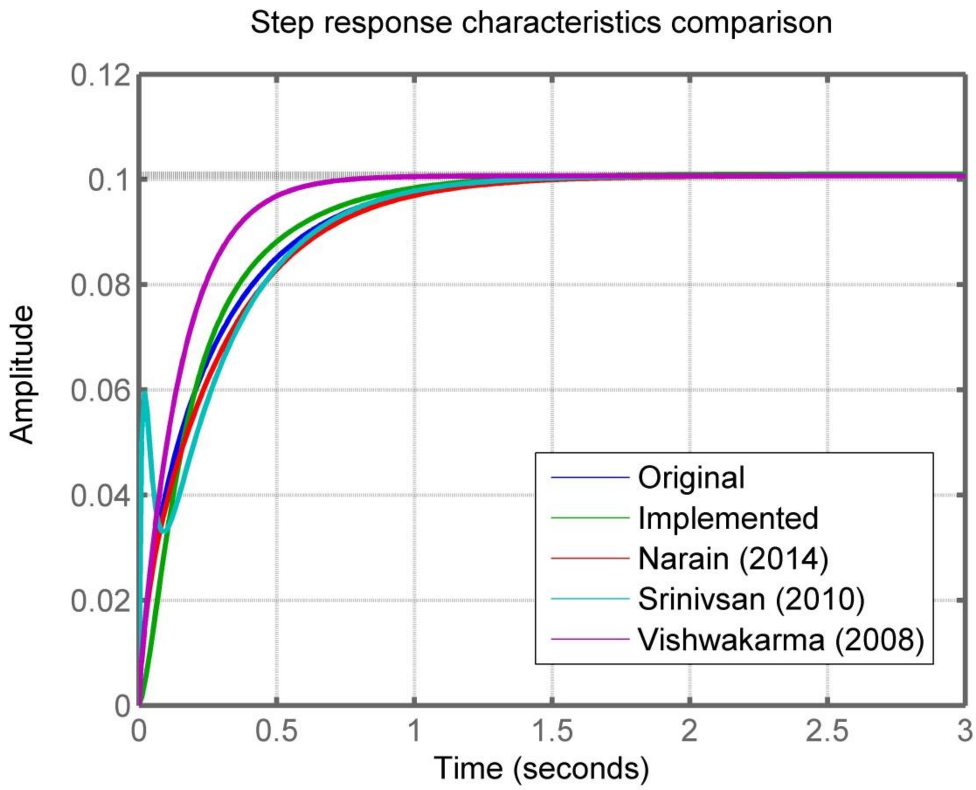


Figure 5.21. Comparative investigation of newly implemented technique with techniques given in the literature for the reduction of 10th order single variable system based on step response characteristics

Comparative investigation: For checking the superiority of the implemented technique over the other techniques studied in the literature, a comparative investigation is performed (Narain et al., 2014; Srinivsan & Krishnan, 2010;

Vishwakarma & Prasad, 2008). All methods are used to reduce the given system of 10^{th} order of equation (5.19), and a system of 3^{rd} order is found out. The comparison is conducted for the system of reduced-order based on characteristics of step response obtained by varied techniques, as depicted in figure 5.21. From the figure, it is analysed that the response obtained by applying the Srinivsan technique shows an initial spike that is of considerable amplitude. The response possessed by Vishwakarma is speedy and reaches the steady-state value earlier than the original system. Furthermore, the step response of the implemented technique and the technique presented by Narain is suitable for the comparative investigation and show similarity with the original system.

Table 5.14. Comparative exploration of errors and time-domain performance parameters, for example 7 for a unit step input

Parameter	ISE $\times 10^{-05}$	ITAE $\times 10^{-03}$	IAE $\times 10^{-03}$	OS (%)	RT (sec.)	ST (sec.)	SSE
Original	-	-	-	0	0.63	1.16	-
Implemented	1.92	1.4	3.5	0	0.53	1.08	0
Narain (2014)	5.54	2.2	2.7	0	0.67	1.20	0.03
Srinivsan (2010)	8.10	1.7	4.9	0	0.65	1.10	0.03
Vishwakarma (2008)	10.6	5.6	10.2	0	0.33	0.60	0.03

A more precise comparative investigation is performed by obtaining time-domain characteristics and error values, as shown in Table 5.14. It is seen from the table that the rise time possessed by implemented technique and the techniques of Narain and Srinivasan are almost similar. Furthermore, the value of settling time for these techniques is also almost similar to the original system. The technique presented by Vishwakarma shows a significant deviation in the rise time and settling time. Moreover, all the techniques excepted implemented technique provides some value of steady-state error. Hence it is advised that the implemented technique provides better values of time-domain performance characteristics. While analysing the error value (ISE, ITAE, IAE), it has resulted that the value of ISE possessed by implemented technique is almost three times lower than that possessed by Narain and lower than other techniques used in the comparative investigation. There is not much difference between ITAE and IAE. Still, the value of ITAE is lowest for implemented technique, and the value of IAE is lowest for the technique presented by Narain. So from the comparative investigation, it is revealed that the technique presented in the research shows better characteristics of performance in comparison to the other techniques used in the comparative investigation, as observed in figure 5.21 and Table 5.14.

From the reduction of the system of 10^{th} order to obtain a system of 3^{rd} order by the method implemented in the research work, it is clear that the method provides a good approximation of the system of higher-order with similar time and frequency domain characteristics and equivalent time moments. Also, the implemented technique provides a stable reduced-order system. Furthermore, the technique implemented here also provides better time-domain characteristics for step input than other techniques presented in the literature.

Example 8: Consider a single variable system of 10^{th} order represented by its transfer function in equation (5.22). The system of 10^{th} order is reduced to a system of 2^{nd} order by the technique implemented in the research work.

$$G_{10}(s) = \frac{N_{10}(s)}{D_{10}(s)} \tag{5.22}$$

$$N_{10}(s) = 5.407e^{19}$$

$$D_{10}(s) = s^{10} + 1800s^{9} + 1.37e^{6}s^{8} + 5.76e^{8}s^{7} + 1.45e^{11}s^{6} + 2.27e^{13}s^{5} + 2.14e^{15}s^{4} + 1.15e^{17}s^{3} + 3.13e^{18}s^{2} + 3.24e^{19}s + 5.407e^{19}$$

The initial step is to obtain the approximation of reduced-order for performing partial fraction expansion of the higher-order equation (5.22) to obtain poles and MDIs of the given system. From partial fraction expansion of equation (5.22), the ten poles of the system are obtained as $\sigma_1 = -2.04$, $\sigma_2 = -18.3$, $\sigma_3 = -50.13$, $\sigma_4 = -95.15$, $\sigma_5 = -148.85$, $\sigma_6 = -205.16$, $\sigma_7 = -257.21$, $\sigma_8 = -298.03$, $\sigma_9 = -320.97$ and $\sigma_{10} = -404.16$. Now, the 10^{th} order system is to be reduced into its 2^{nd} order approximated system. As described in the methodology section, two clusters will be formed from ten poles of the given system of higher-order according to the MDIs order. These two clusters are to be made so that each cluster contains a set of real poles. According to the values of MDI, the first cluster is formed of the pole having the largest MDI, i.e. $\sigma_1 = -2.04$, and hence the cluster centre of the first cluster is obtained as $\sigma_{c1} = -2.04$. The value of cluster centre of second clusters is obtained by placing all remaining poles i.e. $\sigma_2 = -18.3$, $\sigma_3 = -50.13$, $\sigma_4 = -95.15$, $\sigma_5 = -148.85$, $\sigma_6 = -205.16$, $\sigma_7 = -257.21$, $\sigma_8 = -298.03$, $\sigma_9 = -320.97$ and $\sigma_{10} = -404.16$ in the second cluster and solving the equation (4.9). Hence, the second cluster's is obtained as $\sigma_{c2} = -18.3$. So, the combination of the centres of the two clusters and formation of the denominator equation of the 2nd order of the system of reduced-order is described in equation (4.10). The denominator equation reduced-order is obtained as:

$$D_2(s) = s^2 + 20.34s + 37.33$$

Now, the denominator equation of the system of reduced-order is utilised for obtaining the numerator of the system of reduced-order. Considering that the numerator is of 1st order having two unknown coefficients. So, the system of reduced-order is displayed as follows:

$$G_2(s) = \frac{c_1 s + c_2}{s^2 + 20.34s + 37.33} \tag{5.23}$$

The next step is to find the value of unknown coefficients by genetic algorithm (GA). The integral square error in the system of higher order's step response of equation (5.22) and the approximated system of reduced-order(5.23) is minimised by GA, and the value of unknown coefficients, i.e. c_1 and c_2 are obtained. A total of 73 iterations are utilised to get the optimised results. After optimisation, the value of unknown coefficients is obtained as $c_1 = 0.0994$ and $c_2 = 14.8$. By putting the value of the unknown coefficients, the equation for the reduced-order system is procured as follows:

$$G_2(s) = \frac{-1.996s + 37.3}{s^2 + 20.34s + 37.33} \tag{5.24}$$

The system of reduced order's accuracy is obtained in equation (5.24) is checked by its comparison with the higher-order system of equation (5.22) based on the frequency and step response behaviour of the system of higher-order and reduced-order. Figure 5.22 shows the step response behaviour and depicts that the system of higher-order and reduced-order shows the same behavioural characteristics in the whole time range. But, some undershoot occurs in the system of reduced order's response at the starting moment.

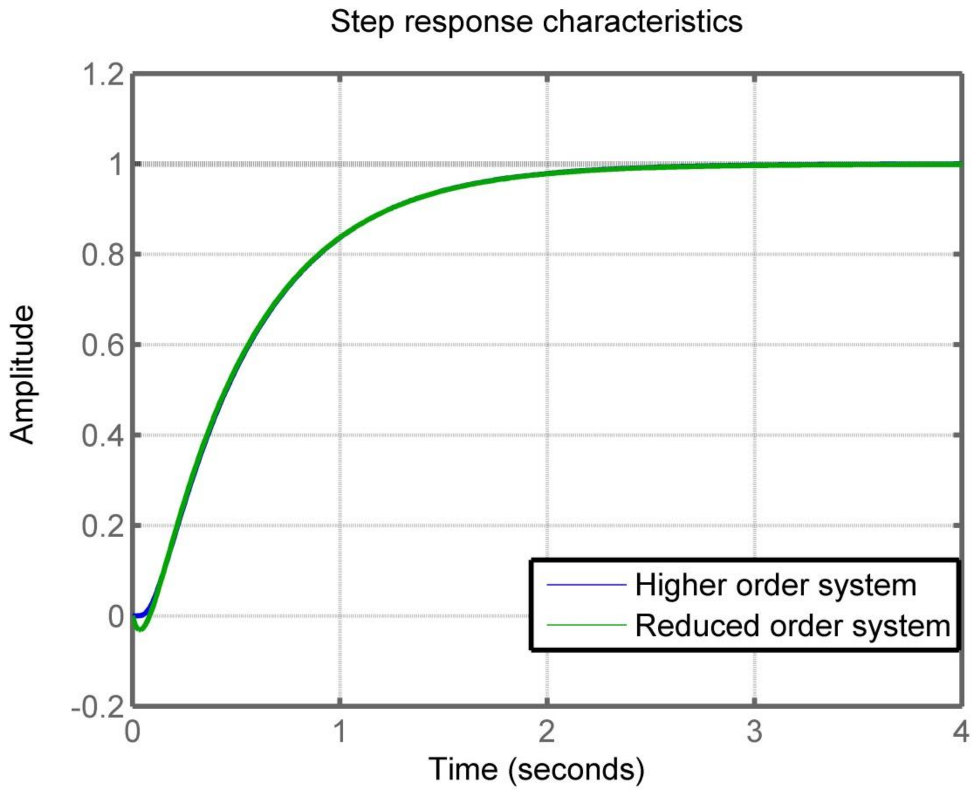


Figure 5.22. System of higher and reduced order's step response characteristics of, for example 8

Moreover, the steady-state value of both systems is seen to be nearly equal to the step response behaviour. The more accurate conclusion on the step response behaviour can be drawn from Table 5.15, which shows that higher and reduced order's rise and settling time values are almost similar. Also, the system of reduced-order has zero overshoot which is similar to the system of higher order's overshoot. Moreover, the obtained system of reduced-order has zero steady-state error for the input of the unit step. While analysing the error in the system of higher and reduced-order, it is found out that the value IAE, ITAE & ISE is shallow. It signifies that there is significantly less error in the system of higher and lower-order. So, the step response comparison in figure 5.22 and Table 5.15 shows that the system of reduced 3rd order's time-domain characteristics matches the system of higher 10th order.

Hence, the system of reduced-order is termed a good approximation of the system of higher-order based on the step response behaviour.

Table 5.15. Value of errors and various parameters to compare higher 10th order system and obtained 2nd order system of example 8 for a unit step input

Parameter	ISE	ITAE	IAE	OS (%)	RT (sec.)	ST (sec.)	SSE	GM	PM
HOS	-	-	-	0	1.08	2.02	-	12.8	180
ROS	6.99×10^{-05}	6.7×10^{-03}	7.0×10^{-03}	0	1.08	2.01	0	10.2	∞

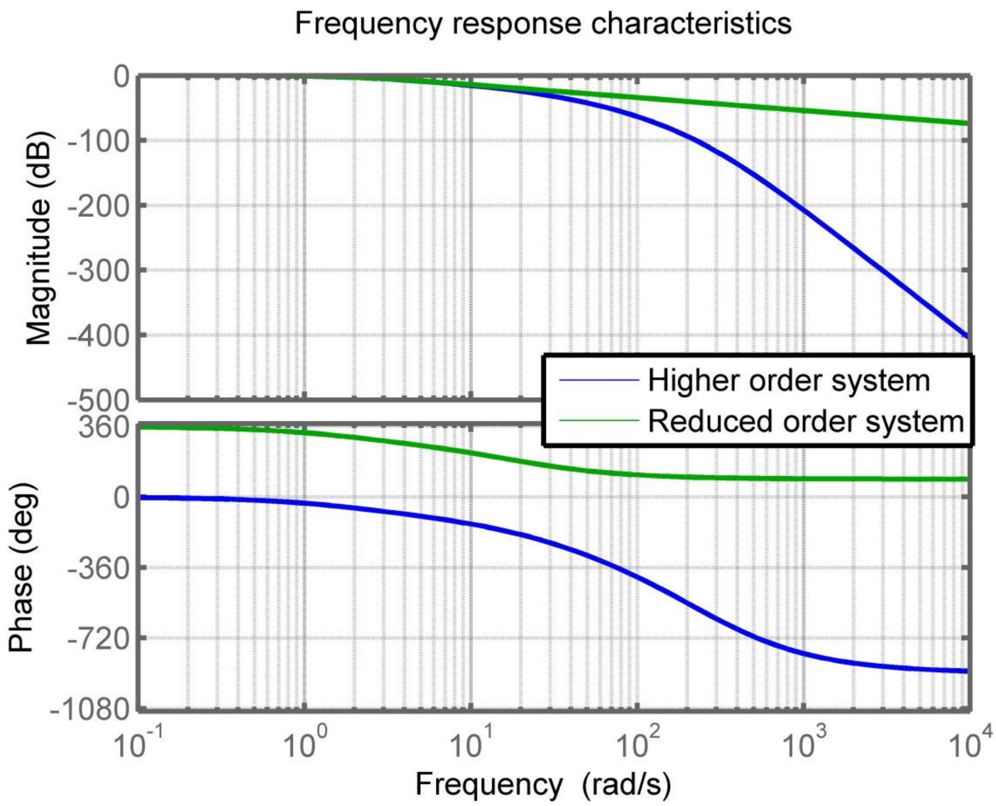


Figure 5.23. System of higher and reduced order's frequency response characteristics, for example 8

While analysing the system of higher and reduced order's frequency response behaviour from figure 5.23, it is seen that both systems' magnitude response is almost the same at low frequencies but shows a change in magnitudes at higher frequencies. Moreover, the phase response of the system of reduced-order has deviated from the system of higher-order with a phase shift of 360º at lower frequencies. Still, the phase difference increase with an increase in frequency. The main characteristic of frequency response, i.e., GM and PM, is noted from Table 5.15, which results that the value of GM is similar for the system of higher and reduced-order and the value of PM reaches infinity after reduction of 10^{th} order system by implemented technique. So, from the higher order's response characteristics of frequency, it is clear that the frequency response varies after reduction, but the value of characteristics parameters does not change.

The equivalency in the system of higher and reduced-order can also be proven by obtaining both system's time moments. The similar value of the time moments of the two systems results that these systems are similar. Both system's first three-time moments are obtained from equation (4.17) by putting the value of $i=1, 2$ and 3. The higher-order system's first three-time moments are $T_{l1}= -0.0599$, $T_{l2}= 0.602$ and $T_{l3}= -0.88$ and the reduced-order system's first three-time moments are $T_{r1}= -0.0597$, $T_{r2}= 0.598$ and $T_{r3}= -0.88$. It shows that the time moments of the system of higher-order and reduced-order obtained by implemented method is approximate. Hence, it results that the system of 10^{th} order is similar to the derived system of 3^{rd} order.

Comparative investigation: For checking the superiority of the implemented technique over the other techniques studied in the literature, a comparative investigation is performed (Al-Dabooni & Wunsch, 2019; Parmar et al., 2007b; J. Singh et al., 2016; Tiwari & Kaur, 2020a). All methods are used to reduce the given 10^{th} order system of equation (5.22), and a reduced system of 2^{nd} order is determined. The basis of comparison is the step response characteristics of the system of reduced-order, which are determined through numerous methods, as shown in figure 5.24. It is studied from the figure that the response obtained by applying all techniques shows an approximately similar response. The starting value of all responses is at the same amplitude. Moreover, all responses tend to show the same value of steady-state error by showing that the path similar to the original system is traversed except the response shown by Tiwari's technique. From the highlighted portion, it is clear that the response obtained by the technique of Parmar shows some value of suddenly occurring undershoot, and this undershoot is higher in the response of Singh. This sudden undershoot is not desirable as it can damage the system and hence not suitable for realistic simulation.

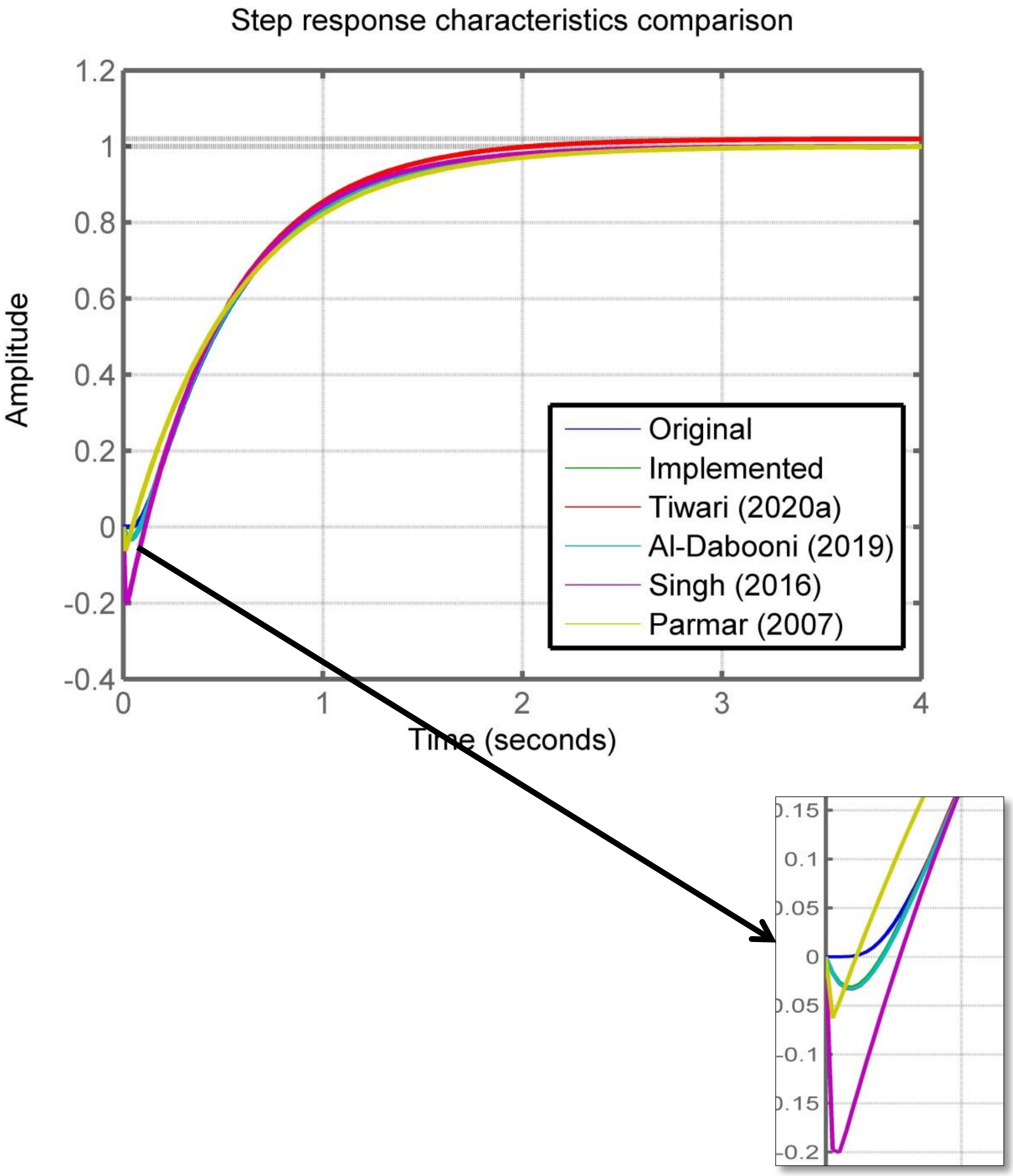


Figure 5.24. Comparative investigation of newly implemented technique with techniques given in the literature for the reduction of 10th order single variable system based on step response characteristics

Furthermore, the implemented technologies and the techniques of Al-dabooni and Tiwari also show some undershoot, which is not much harmful to the system because the change is not sudden. A more precise comparative investigation is performed by obtaining time-domain characteristics and error values, as shown in Table 5.16.

Table 5.16. Comparative exploration of errors and time-domain performance parameters, for example 8 for a unit step input

Parameter	ISE	ITAE	IAE	OS (%)	RT (sec.)	ST (sec.)	SSE
Original	-	-	-	0	1.08	2.02	-
Implemented	6.99×10^{-05}	0.0017	0.0070	0	1.08	2.01	0
Tiwari (2020a)	0.0014	0.1566	0.0733	0	1.08	2.01	0.02
Al-Dabooni (2019)	7.33×10^{-05}	0.0015	0.0050	0	1.08	2.01	0
Singh (2016)	0.0020	0.0149	0.0284	0	1.05	1.89	0
Parmar (2007)	0.0015	0.0409	0.0455	0	1.21	2.18	0

It is seen from the table that the rise time possessed by implemented technique and the techniques of Tiwari and Al-dabooni are almost similar. Furthermore, the value of settling time for these techniques is also almost similar to the original system. The technique presented by Singh and Parmar shows the deviation in the rise time and settling time. Also, all the techniques other than Tiwari show zero value of steady-state error. Hence, the implemented technique and Al-dabooni technique provide better values of time-domain performance characteristics. While analysing the error value (ISE, ITAE, IAE), it has resulted that the value of ISE possessed by implemented technique is lower than all the techniques used in the comparison. The implemented technique and Al-dabooni technique show the least and approximately same value of ISE, ITAE and IAE between all techniques discussed in the comparative investigation. Hence, the overall comparison shows that the implemented technique and Al-dabooni technique possess the better value of all parameters discussed in the comparative investigation. So from the comparative investigation s, it is revealed that the technique presented in the research depicts better performance than the other techniques used in the comparative investigation, as observed in figures 5.24 and Table 5.16.

From the reduction of the 10th order system to obtain a system of 2nd order by the method implemented in the research work, it is seen that the method gives a good approximation of the higher-order system with similar time and frequency domain characteristics and equivalent time moments. Also, the implemented technique provides a stable reduced-order system. Furthermore, the technique implemented here also provides better time-domain characteristics for step input than other techniques presented in the literature.

Example 9: The 48th order model of a Los Angeles university hospital building has eight floors. Two sections on each floor are formed based on the three parameters (x-axis, y-axis and orientation). The model consisted of differential equations of twenty-four orders for each section, resulting in a 48th order system.

After performing the reduction by implemented technique, a system of reduced-order of 10th order is found out. The transfer function of the reduced system of 10th is obtained as equation (5.25):

$$G_{10}(s) = \frac{\begin{array}{c} 0.00943s^9 + 0.0443s^8 + 1.72s^7 + 9.02s^6 - 14.5s^5 - 72s^4 \\ -123s^3 + 68.5s^2 + 62.8s - 24 \end{array}}{\begin{array}{c} s^{10} + 3.7014s^9 + 1054.2s^8 + 2775.9s^7 + 3.3407e05s^6 + 6.095e05s^5 \\ + 4.0332e07s^4 + 4.5365e07s^3 + 1.6536e09s^2 + 9.0437e08s + 2.1204e10 \end{array}} \tag{5.25}$$

The GA utilises a total 65 number of iterations to obtain the optimised value of numerator coefficients of a system of reduced-order. The accuracy of the system of reduced-order obtained in equation (5.25) is checked by its comparison with the system of higher-order of 48th order based on the frequency and step response behaviour of the system of higher and reduced order. Figure 5.25 depicts the step response behaviour, which signifies that the system of reduced-order shows oscillations of higher amplitude than the original system’s response. Still, the response of both systems takes the approximately same time to reach the steady state. The more accurate conclusion on the step response behaviour can be drawn from Table 5.17.

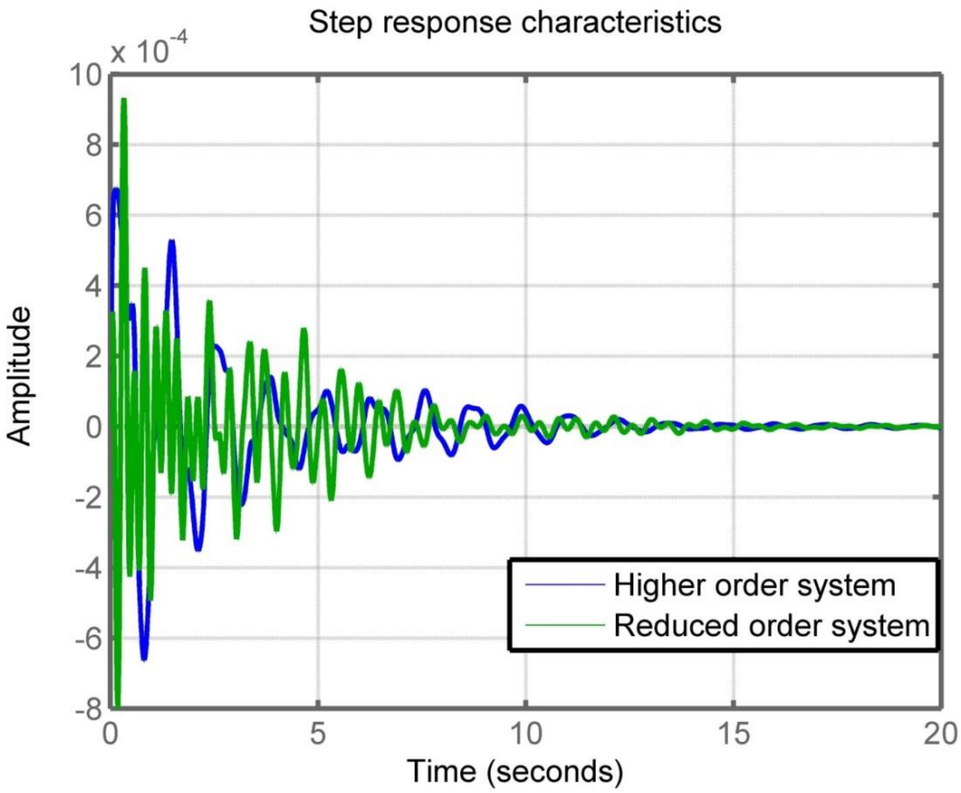


Figure 5.25. System of higher and reduced order's step response characteristics, for example 9

It is visible by the table that the rise and settling time's value for the system of higher-order and reduced-order is almost the same. Furthermore, the overshoot exhibited by the reduced-order system is very low, approximately similar to the response's overshoot of the system of higher-order. Moreover, when the input is the unit step, the error at steady-state error is also noted as zero in the system of reduced-order obtained by implemented technique. While analysing the error in the system of higher and reduced-order, it is found out that the value of ISE, IAE & ITAE is shallow. It signifies that there is significantly less error amidst the system of higher and lower-order. So, it is clear from the step response comparison in figure 5.25 and Table 5.17 that the system of 10^{th} order's time-domain characteristics approximately matches with the higher system of 48^{th} order. Hence, the system of reduced-order is termed a fair approximation of the system of higher-order based on the system's step response behaviour. While analysing the system of higher and reduced order's frequency response behaviour, it is evident from figure 5.26 that both system's magnitude response is almost identical at higher frequencies but showing a change in magnitude at lower frequencies. Moreover, the system of reduced order's phase response has deviated from the system of higher-order at lower frequencies. Still, the phase of the reduced-order system becomes approximately similar to the higher-order system. The main characteristic of frequency response, i.e.GM & PM's, is noted from Table 5.17, which results that GM of the system of higher-order has the same polarity as the system of reduced-order. Also, PM reaches infinity after reduction of 48^{th} order system by implemented technique. So, through the frequency response characteristics, the system of higher order's frequency varies after reduction, but the value of characteristics parameters does not change.

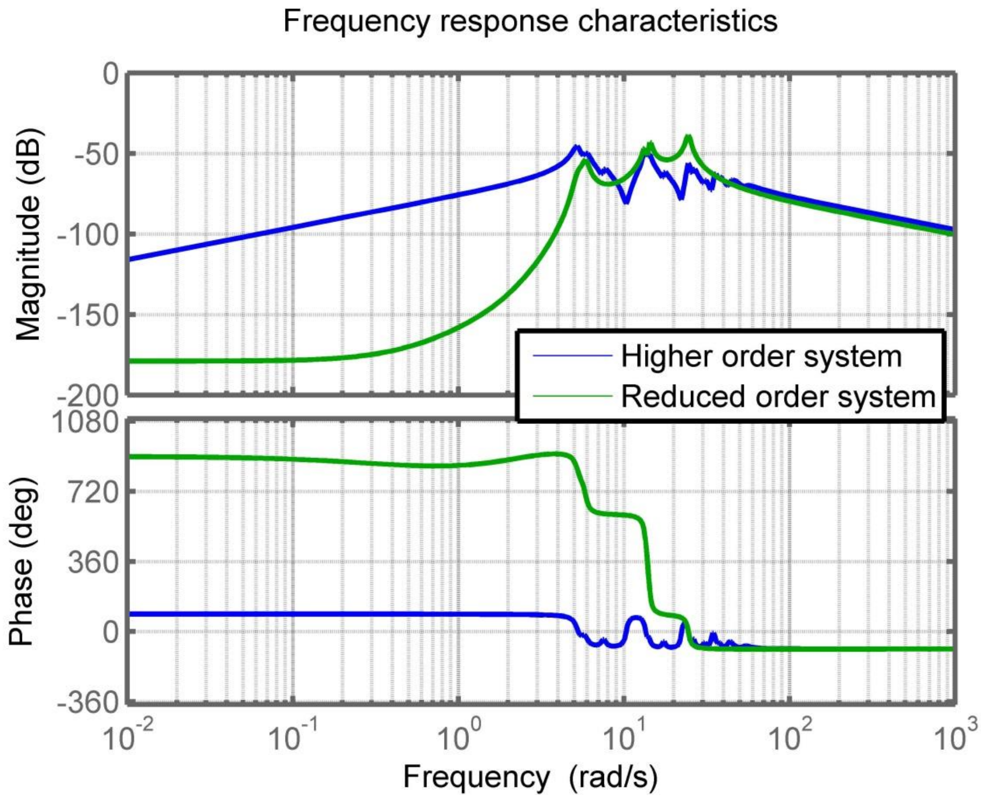


Figure 5.26. System of higher and reduced order's frequency response characteristics, for example 9

The equivalency in the system of higher and reduced-order can be proven by obtaining both systems' time moments. The similar value of the time moments of the two systems results that these systems are similar. Both systems' first three time-moments are obtained from equation (4.17) by putting the value of $i=1$, 2 and 3. The higher-order system's first three time-moments are $T_{l1}= 1.58\times10^{-06}$, $T_{l2}= 4.4842\times10^{-06}$ and $T_{l3}= -1.962\times10^{-05}$, and the system of reduced order's first three time-moments are $T_{r1}= 3\times10^{-09}$, $T_{r2}= 6.38\times10^{-09}$ and $T_{r3}= -3.7\times10^{-08}$. It shows that the system of higher and reduced order's time moments obtained by implemented method is approximately the same. Hence, it results that the 48th order system is similar to the derived 10th order system.

Table 5.17. Value of errors and various parameters to compare higher 48th order system and obtained 10th order system of example 9 for the unit step input

Parameter	ISE	ITAE	IAE	OS (%)	RT (sec.)	ST (sec.)	SSE	GM	PM
HOS	-	-	-	∞	0	12.95	-	∞	∞
ROS	9.47×10^{-09}	1.1×10^{-03}	0.2×10^{-03}	7.05×10^{07}	7.804×10^{-05}	13.70	0	140.9	∞

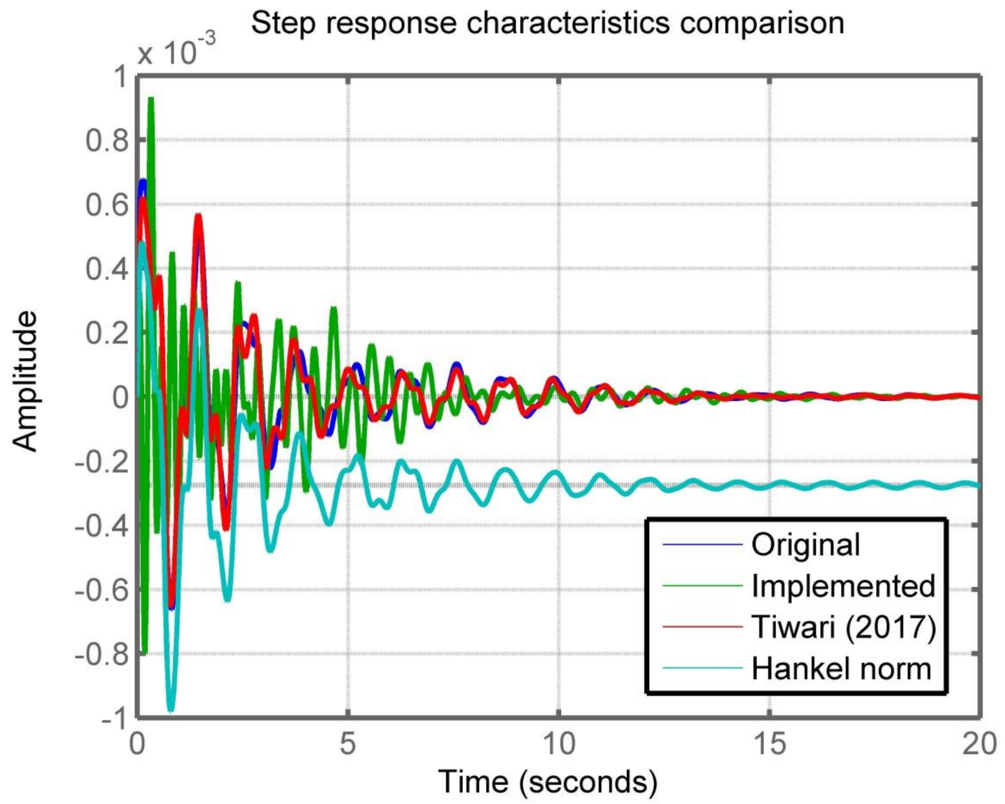


Figure 5.27. Comparative analysis of newly implemented technique with techniques given in the literature for the reduction of 48th order single variable system based on step response characteristics

Comparative investigation: For checking the superiority of the implemented technique over the other techniques studied in the literature, a comparative analysis is performed (Glover, 1984; Tiwari & Kaur, 2017). All methods are employed to reduce the given system of the 48^{th} order and obtain a reduced system of the 10^{th} order. The comparison is based on the system of reduced order's step response characteristics obtained by varied techniques, as depicted in figure 5.27. From the figure, it is analysed that the response obtained by applying all techniques shows an approximately similar response. The starting value of all responses is at the same amplitude. Moreover, all responses tend to show the exact value of steady-state error by traversing a similar path to the original system. Implemented techniques' response shows some extra overshoot than the original system but enters the steady-state simultaneously. A more precise comparative investigation is performed by procuring time-domain characteristics and error values, as shown in Table 5.18. It clears that the rise time possessed by implemented technique is approximately similar to the original system and has better value than other techniques.

Furthermore, the value of settling time for implemented technique shows a deviation from the original system, but the percentage overshoot value decreases in the implemented technique. Also, the value of the steady-state error in the system of reduced and higher-order is zero for almost all techniques. So, the overall comparison results that implemented technique shows similarity with the original system for most of the parameters. Hence it is advised that the implemented technique provides better values of time-domain performance characteristics. While analysing the error value (ISE, ITAE, IAE), it has resulted that the value of ISE possessed by implemented technique is lower than all the techniques used in the comparison. The implemented technique and Tiwari's technique show the least and approximately same value of ISE, ITAE and IAE between all techniques discussed in comparative analysis. Hence, the overall comparison shows that the implemented technique possesses the better value of all parameters discussed in the comparative analysis. So from the comparative analysis, it is revealed that the technique presented in the research shows performance characteristics that are better compared to the other techniques used in the comparative investigation, as observed in figure 5.27 and Table 5.18.

From the reduction of a system of 48^{th} order to obtain a system to 10^{th} order by the method implemented in the research work, it is clear that the method provides a fair approximation of the system of higher-order with similar time and frequency domain characteristics equivalent time moments. Also, the implemented technique provides a stable reduced-order system. Furthermore, the technique implemented here also provides better time-domain characteristics for step input than other techniques presented in the literature.

Table 5.18. Comparative analysis of errors and time-domain performance parameters, for example 9 for the unit step input

Parameter	ISE	ITAE	IAE	OS (%)	RT (sec.)	ST (sec.)	SSE
Original	-	-	-	∞	0	12.95	-
Implemented	9.47×10^{-09}	0.0011	0.0002	7.05×10^{07}	7.804×10^{-05}	13.70	0
Tiwari (2017)	2.03×10^{-08}	0.0015	0.0003	∞	0.01	12.77	0
Hankel norm	1.52×10^{-06}	0.0551	0.0055	2.54×10^{02}	0.05	12.67	0

5.2 REDUCING DISCRETE-TIME SYSTEMS

The methodology implemented in the research work is also applied to the higher-order discrete-time systems, and a reduced-order approximation is obtained. The reduced system procured from the implemented technique is compared with the higher-order system based on the step response and frequency response characteristics.

Example 1: Consider a 4^{th} order single variable system of discrete-time, with the following transfer function:

$$G_4(z) = \frac{0.216608 + 0.319216z - 0.40473z^2 + 0.0547377z^3}{0.282145 - 0.551205z + 0.875599z^2 - 1.36078z^3 + z^4} \quad (5.26)$$

To reduce the systems discrete-time, initially, there is a requirement to convert the system of discrete-time in the corresponding continuous-time system using any transformation as discussed in methodology. By using the bilinear transformation, i.e. $z=s+1$, the corresponding continuous-time transfer function of equation (5.27) is obtained from the discrete-time higher-order system (5.26).

$$G_4(s) = \frac{0.0547377s^3 - 0.2505169s^2 - 0.326039s - 0.2473843}{s^4 + 2.63922s^3 + 2.793259s^2 + 1.11766s + 0.245759} \tag{5.27}$$

The aim is to reduce the 4th order system obtained in equation (5.17) in the 2nd order approximated system by following the implemented methodology. From partial fraction expansion of equation (5.27), the four poles of the system are obtained as $\sigma_{1,2} = -1.0703 \pm 0.6510i$ and $\sigma_{3,4} = -0.2493 \pm 0.3073i$. According to the order of MDIs, there will be two clusters in the system of reduced-order, as depicted in methodology, that is to be formed from four poles of a given higher-order system. As there are only complex poles, the cluster centre of the two clusters is complex conjugate, obtained from equations (4.11) and (4.12) as $\sigma_{c1,2} = -0.291 \pm 0.3699i$. So, combining the two cluster centres and forming the 2nd order denominator equation of the system of reduced-order as described in equation (4.13). Denominator equation of the system of reduced-order is obtained as:

$$D_2(s) = s^2 + 0.5832s + 0.2219$$

Now, the denominator equation of the system of reduced-order is utilised for obtaining the system of reduced order's numerator. Considering that the numerator is of 1st order having two unknown coefficients. So, the system of reduced-order is displayed as follows:

$$G_2(s) = \frac{m_1 s + m_2}{s^2 + 0.5832s + 0.2219} \tag{5.28}$$

The next step is to find the value of unknown coefficients by genetic algorithm (GA). The ISE amidst the higher order's step response equation (5.27) and the approximated system of reduced-order (5.28) is minimised by GA, and the value of unknown coefficients, i.e. m_1 and m_2, are obtained. A total of 82 iterations are utilised to get the optimised results. After optimisation, the value of unknown coefficients is obtained as $m_1 = 0.111$ and $m_2 = -0.223$. By putting the value of the unknown coefficients, the equation for the reduced-order system is procured as follows:

$$G_2(s) = \frac{0.111s - 0.223}{s^2 + 0.5832s + 0.2219} \tag{5.29}$$

Now from the inverse transformation, i.e. *s=z-1*, the continuous-time is converted back to a discrete-time system. So after inverse transformation of the continuous-time system of equation (5.29), the system of discrete-time with reduced-order is obtained as follows:

$$G_2(z) = \frac{0.111z - 0.334}{z^2 - 1.417z + 0.6387} \tag{5.30}$$

So from the implemented technique, the reduced-order approximation of the higher-order system of equation (5.25) is obtained and shown in equation (5.30). The accuracy of the system of reduced-order, which is found in equation (5.30), is checked by its comparison with the higher-order system of equation (5.25) based on the system of higher and reduced order's frequency and step response. Figure 5.28 shows the step response, which shows that the system of higher-order and reduced-order shows approximately the same behavioural characteristics in the whole time range. But, there is some extra value of overshoot and undershoot occurring in the response of the reduced-order system. But, for both the systems, the value of rise time and steady-state is found out to be nearly equal to the step response behaviour.

Moreover, the more accurate conclusion on the step response behaviour can be drawn from Table 5.19, which depicts that the value of rise time and for the system of higher- and reduced-order is almost similar, but having a slight deviation in settling time. The system of reduced-order settles earlier than the system of higher-order. Also, it is noted that the overshoot exhibited by the system of reduced-order is a little higher than the system of higher order's overshoot in the response. Moreover, for the input of the unit step, steady-state error's value is also noted as zero in the system of reduced-order obtained by implemented technique. While analysing the error in the system of higher and reduced-order, it is found out that the value of ISE, ITAE and IAE is shallow. It signifies that there is significantly less error in between the system of higher and lower-order. So, the step response comparison in figure 5.28 and Table 5.19 shows that the characteristics in the time domain of the reduced discrete-time system of 2^{nd} order approximately match with the higher 4^{th} order discrete-time system.

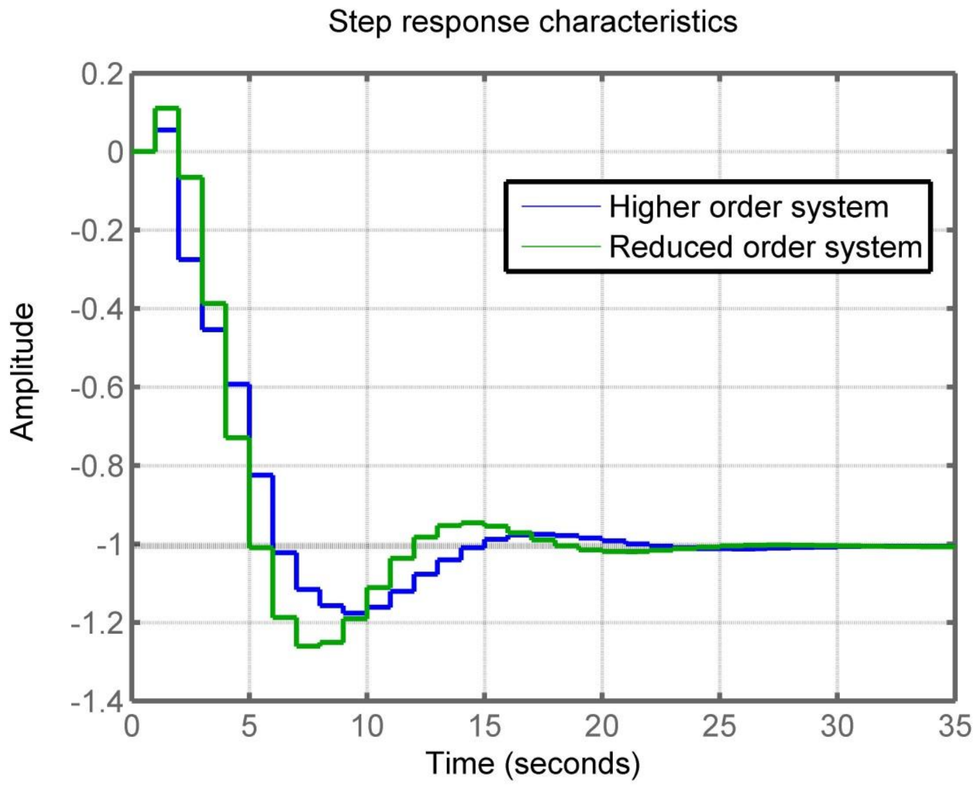


Figure 5.28. Discrete-time system of higher and reduced order's step response characteristics for example 1

Table 5.19. Value of errors and various parameters to compare higher 4th order discrete-time system and obtained 2nd order discrete-time system of example 1 for the unit step input

Parameter	ISE	ITAE	IAE	OS (%)	RT (sec.)	ST (sec.)	SSE	GM	PM
HOS	-	-	-	16.87	4	20	-	0.99	-114
ROS	0.019	1.395	0.166	25.35	4	17	0	0.99	-173

While analysing the system of discrete-time of higher and reduced order's frequency response behaviour shown in figure 5.29, it is clear that both the system's magnitude response is almost the same at low frequencies but shows a change in magnitude at higher frequencies. Moreover, the reduced-order system removes the peak in magnitude response of the system of higher-order. The system of reduced order's phase response is also similar to the system of higher-order. The main characteristic of frequency response, i.e. the GM and PM's value, is found in Table 5.19. The value of GM of higher and reduced-order systems is almost similar, but a slight deviation in the value of PM is obtained. So, from the characteristics of frequency response, it is found out that the frequency response of the discrete-time system of higher-order is almost similar to the discrete-time system of reduced-order.

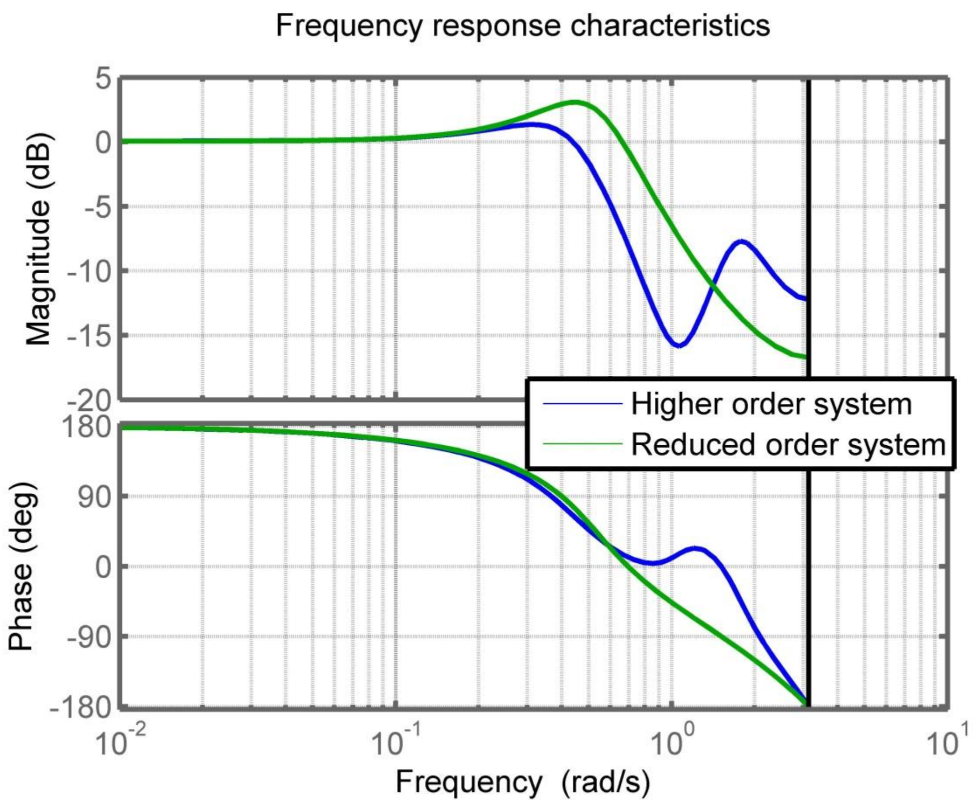


Figure 5.29. System of discrete-time of higher and reduced order's frequency response characteristics for example 1

The equivalency in the system of higher and reduced-order can also be proven by obtaining both systems' time moments. The similar value of the time moments of the two systems results that these systems are similar. Both systems' first three time-moments are obtained from equation (4.17) by putting the value of *i=1, 2* and *3*. The system of higher order's first three-time moments are T_{l1}= *-0.3684,* T_{l2}= *-3.456* and T_{l3}= *-9.51* and the system of reduced order's first three time-moments are

T_{r1}= *-0.9864*, T_{r2}= *-2.739* and T_{r3}= *-8.965*. It shows that the system of higher-order and reduced-order time moments are obtained by implemented method is approximately the same. Hence, it results that the 4th order original system is similar to the derived 2nd order system.

Comparative investigation: For checking the superiority of the implemented technique over the other techniques studied in the literature, a comparative analysis is performed (Hsieh & Hwang, 1990; Narwal & Prasad, 2017; Satakshi et al., 2005; C. N. Singh et al., 2019). All methods are used to reduce the given 4th order discrete-time system, and a 2nd order reduced-order system with discrete-time is obtained. The comparison is conducted based on the step response characteristics of the system of reduced-order obtained by varied methods, as depicted in figure 5.30. It shows that the response obtained by applying all techniques shows approximately similar responses except the technique presented by Hsieh and Satakshi. The starting value of all responses is at the same amplitude. The response of implemented technique and techniques of Singh and Narwal traverses a similar path as traversed by the original system. Moreover, all responses except Hsieh and Satakshi tend to show the exact value of steady-state error by following the track similar to the original system. A more precise comparison study is carried out by obtaining the value of time-domain characteristics and error values in Table 5.20. It clears that the rise time possessed by implemented technique and Narwal's method is the nearest method to the original one and has better value than other techniques. But, the technique of Narwal provides a significant difference in settling time.

Furthermore, the value of settling time for implemented technique shows a slight deviation from the original system. The technique presented by Satakshi possess the most accurate value of settling time, but other parameters show a significant deviation from the original system. The value of percentage overshoot is best for the technique of Singh and Narwal, but the value of settling and rise time makes the techniques unsuitable. Also, the method of Narwal provides a small amount of steady-state error, which is not desirable. So, the overall comparison results that implemented technique shows similarity with an original system for error at steady-state, rise time and settling time. Hence it is advised that the implemented technique provides better values of performance characteristics in the time domain. While analysing the values (ISE, ITAE, IAE), it has resulted that the value of ISE possessed by implemented technique is lower than all the techniques used in the comparison. The implemented technique shows the most negligible and approximately the exact value of ISE, ITAE and IAE between all techniques discussed in comparative analysis. Hence, the overall comparison shows that the implemented technique possesses the better value of all parameters discussed in the comparative analysis. So from the comparative analysis, it is revealed that the technique presented in the research shows comparatively good characteristics of performance in comparison to the other techniques used in the comparative analysis as observed in figure 5.30 and Table 5.20.

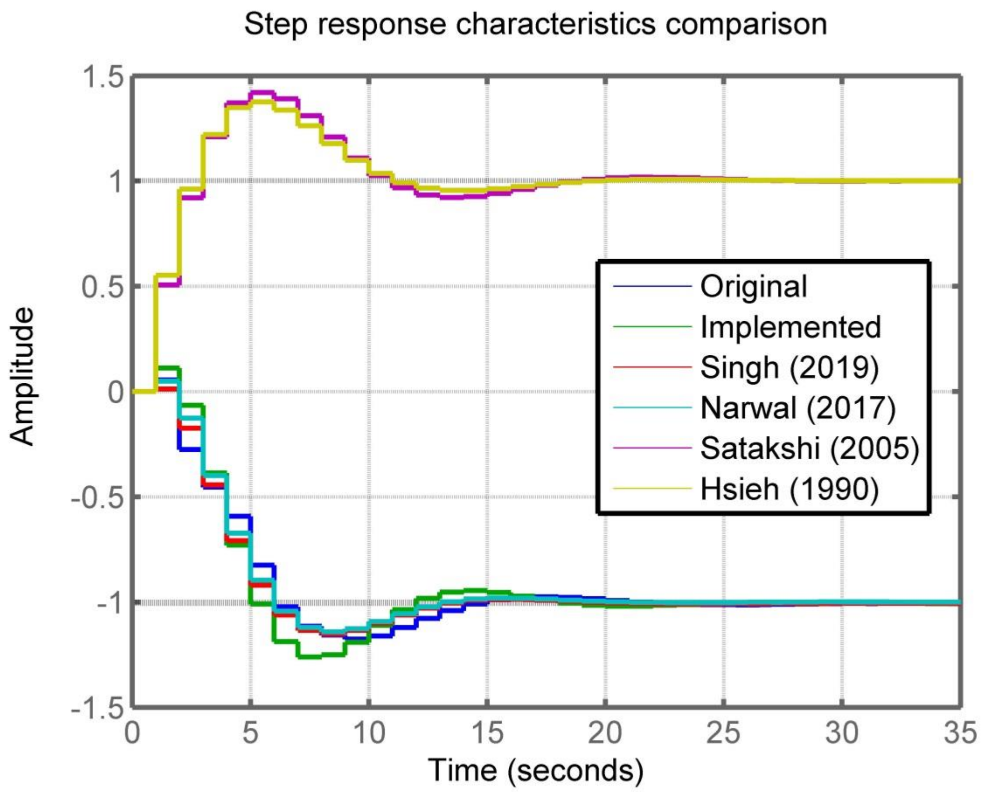


Figure 5.30. Comparative analysis of newly implemented technique with techniques studied earlier for reducing the system of discrete-time with 4th order single based on step response characteristics

From the reduction of the 4th order discrete-time system to procure a 2nd order system through the method implemented in the research work, it is clear that the method provides a relatively good estimate of the system of higher-order with approximately similar time and frequency domain characteristics and equivalent time moments. Furthermore, the technique implemented here also provides better time-domain characteristics for step input than other techniques presented in the literature.

Table 5.20. Comparative analysis of errors and time-domain performance parameters for the discrete-time system of example 1 subjected to the unit step input

Parameter	ISE	ITAE	IAE	OS (%)	RT (sec.)	ST (sec.)	SSE
Original	-	-	-	16.87	4	20	-
Implemented	0.0196	1.395	0.1666	25.35	4	17	0
Singh (2018)	0.0505	6.805	0.8010	14.47	3	13	0
Narwal (2017)	0.0554	8.7771	0.8667	14.23	4	13	0.006
Satakshi (2005)	140.6	1271.4	69.191	41.73	1	18	0.004
Hsieh (1990)	140.32	1271.4	69.220	37.33	1	17	0.005

Example 2: A system with discrete-time possessing a single variable with 6th order is considered and is represented by the transfer function as follows:

$$G_6(z) = \frac{0.3277z^6 + 0.9195z^5 + 1.038z^4 + 0.5962z^3 + 0.1618z^2 + 0.00698z - 0.005308}{z^6 + 1.129z^5 + 0.2889z^4 - 0.08251z^3 - 0.04444z^2 - 0.00476z} \tag{5.31}$$

For reducing the systems with discrete-time, the initial requirement is to convert the system of discrete-time in the corresponding continuous-time system using any transformation as discussed in methodology. Using the Bilinear Tustin transformation, i.e. $z = \frac{1 + sT/2}{1 - sT/2}$ for 1 second, the corresponding continuous-time transfer function of equation (5.32) is obtained from the discrete-time higher-order system (5.31) as follows:

$$G_6(s) = \frac{s^5 + 15.6s^4 + 124.2s^3 + 510.3s^2 + 1166s + 959.3}{s^6 + 21s^5 + 175s^4 + 735s^3 + 1624s^2 + 1764s + 720} \tag{5.32}$$

The aim is to reduce the 6th order system obtained in equation (5.32) in the 3rd order approximated system by following the implemented methodology. From partial fraction expansion of equation (5.32), the system's six poles are obtained as: $\sigma_1 = -1$, $\sigma_2 = -2$, $\sigma_3 = -3$, $\sigma_4 = -4$, $\sigma_5 = -5$ *and* $\sigma_6 = -6$. There will be three clusters in the system of reduced-order as presented in methodology that is to be formed from six poles of a given higher-order system according to the order of MDIs. As there are only real poles, so the clusters are formed of only real poles. The first cluster contains the highest MDI pole, i.e. $\sigma_4 = -4$, forming the cluster centre. The second cluster is formed of the pole having the second largest value of MDI, i.e. $\sigma_5 = -5$, and hence forming the second cluster's centre. All the remaining poles form the third cluster, and hence the third cluster's centre is obtained from equation (5.9) as $\sigma_{c3} = 1.2457$. So, combining the three cluster centres and forming the 3rd order denominator equation of system of reduced-order depicted through equation (5.10). The denominator equation of reduced-order is obtained as:

$$D_3(s) = s^3 + 10.25s^2 + 31.21s + 24.91$$

Now, the denominator equation of the system of reduced-order is utilised for obtaining the numerator of the reduced-order system. Considering that the numerator is of 2nd order having three unknown coefficients. Hence, the system of reduced-order with the unknown numerator is displayed as follows:

$$G_3(s) = \frac{m_1 s^2 + m_2 s + m_3}{s^3 + 10.25s^2 + 31.21s + 24.91} \qquad (5.33)$$

The next step is to find the value of unknown coefficients by genetic algorithm (GA). The integral square error amidst the system of higher order's step response equation (5.32) and the approximated system of reduced-order (5.33) is minimised by GA, and the value of unknown coefficients, i.e. m_1, m_2 and m_3, are procured. A total of 182 iterations are utilised to get the optimised results. After optimisation, the value of unknown coefficients is obtained as $m_1 = 1.8$, $m_2 = 2.75$ and $m_3 = 33$. By putting the value of the unknown coefficients, the equation for the reduced-order system is procured as follows:

$$G_3(s) = \frac{1.8s^2 + 2.75s + 33}{s^3 + 10.25s^2 + 31.21s + 24.91} \qquad (5.34)$$

Now from the inverse transformation, i.e. $s = \frac{2}{T} \cdot \frac{1 - z^{-1}}{1 + z^{-1}}$, the continuous-time system is converted back to a discrete-time system. So after inverse transformation of the continuous-time system of equation (5.34), the system of discrete-time with reduced-order is obtained as follows:

$$G_3(z) = \frac{45.7z^3 + 97.3z^2 + 86.3z + 36.2}{136.3z^3 + 72.15z^2 - 4.69z - 4.51} \quad (5.35)$$

So from the implemented technique, the reduced-order approximation of the system of higher-order of equation (5.31) is procured, and equation (5.35) depicts it. The accuracy of the system of reduced-order procured from equation (5.35) is checked by its comparison with the higher-order of equation (5.31) based on both systems' frequency and step response behaviour. Figure 5.31 shows the step response behaviour, which shows that the system higher reduced-order shows approximately the same behavioural characteristics in the whole time range. But, there is a little value of overshoot occurring in the response of the system of reduced-order. But, both systems' rise time and steady-state value are seen to be nearly equal from the step response behaviour.

Table 5.21. Value of errors and various parameters to compare higher 6th order discrete-time system and obtained 3rd order discrete-time system of example 2 for the unit step input

Parameter	ISE	ITAE	IAE	OS (%)	RT (sec.)	ST (sec.)	SSE	GM	PM
HOS	-	-	-	0.026	2	4	-	41.5	129
ROS	2.6 $\times 10^{-05}$	0.666	0.119	0.591	2	5	0	42.6	126

Moreover, the more accurate conclusion on the step response behaviour can be drawn from Table 5.21 which shows that both for a system of higher and lower order rise time's value is almost similar, but having a slight deviation in settling time. Also, it is noted that the overshoot exhibited by the system of reduced-order is a little more than the system of higher order's overshoot in the response.

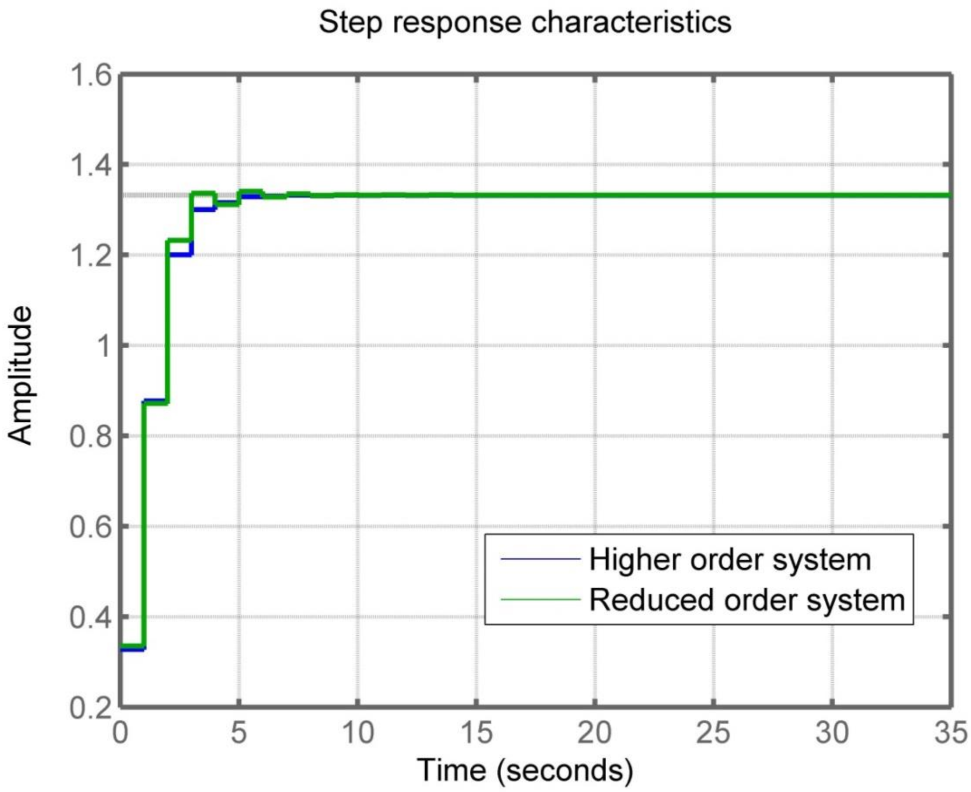


Figure 5.31. Discrete-time system of higher and reduced order's step response characteristics, for example 2

Moreover, for input equal to the unit step, the error at steady state is also noted as zero in the system of reduced-order obtained by implemented technique. While analysing the error in the system of higher and reduced-order, it is observed that the value of ITAE, IAE and ISE is shallow. It signifies that there is significantly less error in between the system of higher and lower-order. So, figure 5.31 and Table 5.21 shows that the time-domain characteristics of the reduced 3rd order discrete-time system approximately match with the higher 6th order discrete-time system. While analysing the system of higher and reduced order's frequency response behaviour in a system with discrete-time, figure 5.32 clarifies that both systems' magnitude response is almost the same over a complete frequency range. Furthermore, the system of reduced order's phase response is also found to be similar to the system of higher-order. The main characteristic of frequency response, i.e. the value of GM & PM, is procured from Table 5.21, which results that the value of GM and PM of the higher and reduced-order system is approximately the same. So, from the characteristics of frequency response, it results that the frequency response of the discrete-time system with higher-order is almost similar to the reduced-order discrete-time system.

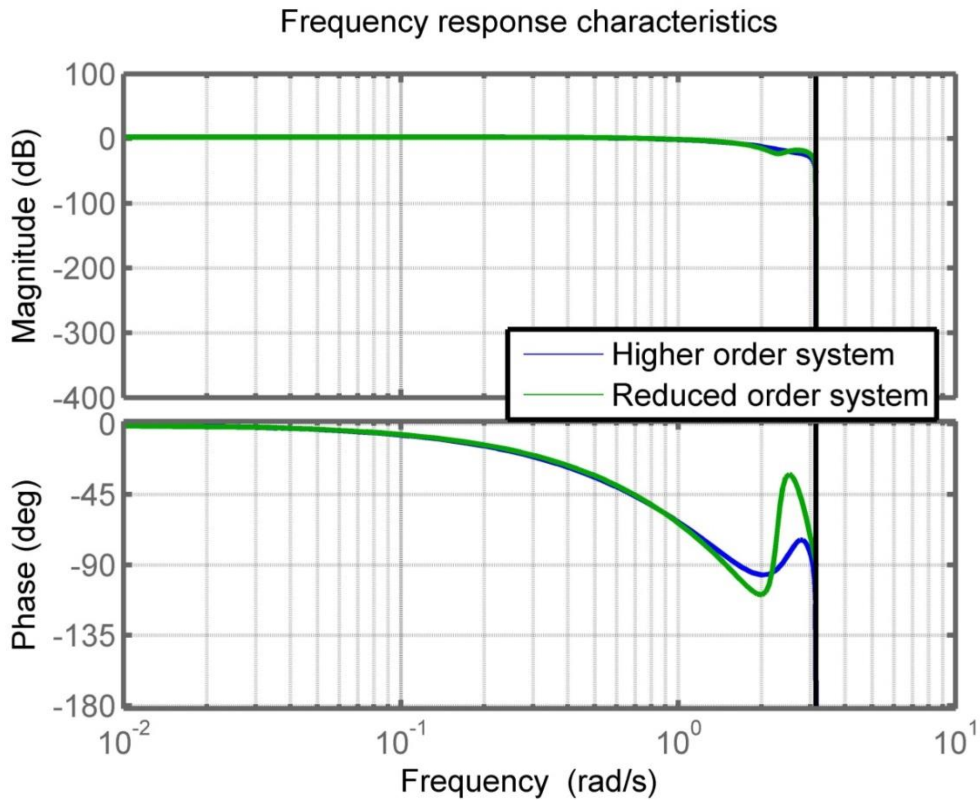


Figure 5.32. System with discrete-time with higher and reduced order's frequency response characteristics, for example 2

The equivalency in the system of higher and reduced-order can also be proven by obtaining both systems' time moments. The similar value of the time moments of the two systems results that these systems are similar. The first three time-moments of both systems are obtained from equation (4.17) by putting $i=1, 2$ and 3. The discrete-time system with higher order's first three time-moments are $T_{ll1}= -1.6468$, $T_{ll2}= 3.4668$, and $T_{ll3}= -10.3464$ and the system of reduced order with discrete time's first three time-moments are $T_{rl1}= -1.5494$, $T_{rl2}= 3.1669$ and $T_{rl3}= -9.5326$. It shows that the system of higher and reduced order's time moments are obtained by implemented method is approximately the same. Hence, the 6th order discrete-time system is similar to the derived 3rd order discrete-time system.

Comparative investigation: For checking the superiority of the implemented technique over the other techniques studied in the literature, a comparative analysis is performed (Glover, 1984; Moore, 1981; Narain et al., 2013; Nasiri Soloklo et al., 2015). All methods are used to reduce the given 6^{th} order discrete-time system, and a 3^{rd} order reduced system with discrete-time is obtained. The step response is the basis of the comparison of the system of reduced-order obtained by various techniques, as shown in figure 5.33.It is analysed from figure 5.33 that the response obtained by applying all techniques shows an approximately similar response. The starting value of all responses is at the same amplitude except for balanced truncation. It is clear from the figure that the significant deviation is seen in the reduced-order system of the balanced truncation technique.

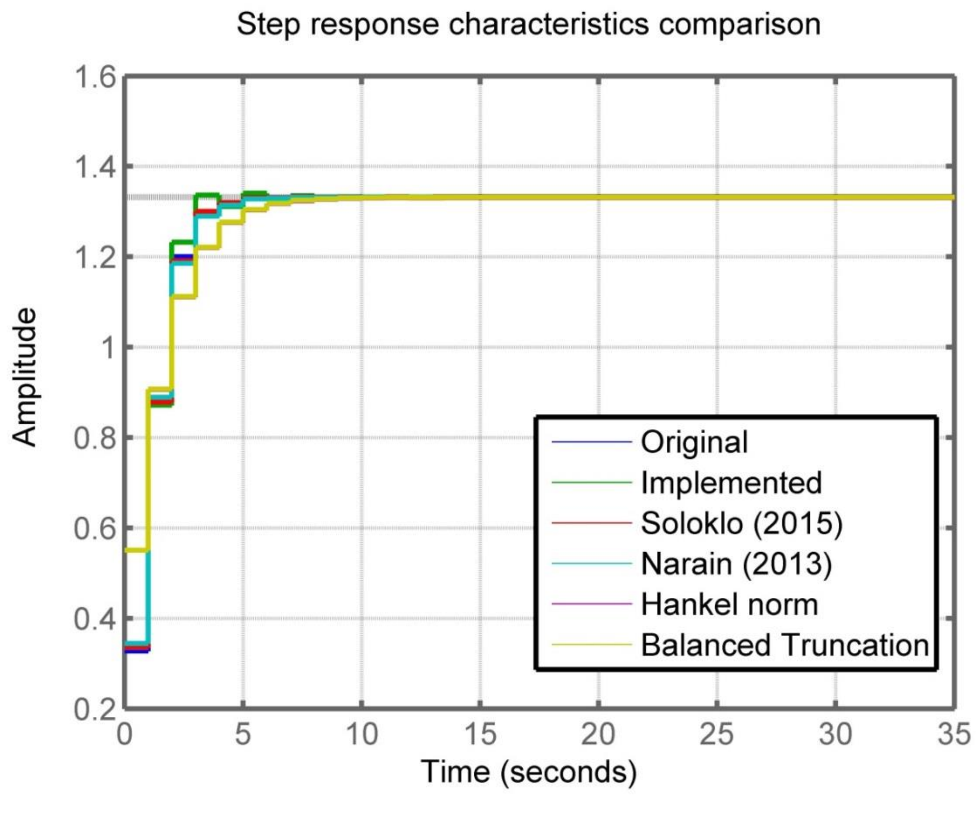


Figure 5.33. Comparative analysis of newly implemented technique with techniques studied earlier for reducing the 6^{th} order single variable discrete-time system based on step response characteristics

More precisely, the comparative analysis is performed by obtaining the value of characteristics in the time domain and error values, as shown in Table 5.22. The rise time possessed by implemented technique and technique of Soloklo and Narain is most similar to the original system. It has a better value compared to Hankel norm approximation and balanced truncation. Also, the value of settling time for the technique of Soloklo and Narain is approximately similar to implemented technique and original system. The value of percentage overshoot has deviated for all techniques. Also, the technique of Hankel norm approximation provides a small amount of steady-state error, which is not desirable. So, the overall comparison results that implemented technique shows similarity with an original system for error at steady state, settling time and rise time. Hence it is advised that the implemented technique provides better values of time-domain performance characteristics.

Table 5.22. Comparative analysis of errors and time-domain performance parameters for a discrete-time system of example 2 subjected to the unit step input

Parameter	ISE	ITAE	IAE	OS (%)	RT (sec.)	ST (sec.)	SSE
Original	-	-	-	0.026	2	4	-
Implemented	2.6×10^{-05}	0.0666	0.0119	0.059	2	5	0
Soloklo (2015)	1.8×10^{-04}	0.460	0.0450	0.010	2	4	0
Narain (2013)	7.3×10^{-04}	0.159	0.060	0.0002	2	4	0
Hankel norm	0.0672	1.491	0.538	0	3	6	0.001
Balanced Truncation	0.0671	1.015	0.5143	0	3	6	0

While analysing the error value (ISE, ITAE, IAE) from Table 5.22, it has resulted that the value of ISE possessed by implemented technique is lower than all the techniques used in the comparison. The implemented technique shows the most negligible and approximately the exact value of ISE, ITAE and IAE between all techniques discussed in comparative analysis. Hence, an overall comparison shows that the implemented technique possesses the better value of all parameters discussed in the comparative analysis. So from the comparative analysis, it is revealed that the technique presented in the research shows better performance compared to the other techniques used in the comparative analysis as observed in figure 5.33 and Table 5.22.

From the reduction of the system of discrete-time having and order of 6 to procure a system of 4^{th} order by the IPCG method, it is clear that the method provides a fair approximation of the system of higher-order with approximately similar time and frequency domain characteristics and equivalent time moments. Also, the implemented technique provides a stable reduced-order system. Furthermore, the technique implemented here also provides better time-domain characteristics for step input than other techniques presented in the literature.

Example 3: Consider a single variable discrete-time system of 8^{th} order described in lower and upper bound. The system of 8^{th} order's transfer function is written as below:

$$G_8(z) = \frac{\begin{array}{c}[1.6484,1.7156]\cdot z^7+[1.0937,1.1383]\cdot z^6+[-0.2142,-0.2058]\cdot z^5 \\ +[0.1490,0.1550]\cdot z^4+[-0.5263,-0.5057]\cdot z^3+[-0.2672,-0.2568]\cdot z^2 \\ +[0.0431,0.0449]\cdot z+[-0.0061,-0.0059]\end{array}}{\begin{array}{c}[23.52,24.48]\cdot z^8+[-1.7156,-1.6484]\cdot z^7+[-1.1383,-1.0937]\cdot z^6+ \\ [0.2058,0.2142]\cdot z^5+[-0.1550,-0.1490]\cdot z^4+[0.5057,0.5263]\cdot z^3+ \\ [0.2568,0.2672,]\cdot z^2+[-0.0449,-0.0431]\cdot z+[0.0059,0.0061]\end{array}} \quad (5.36)$$

Initially, it is required to convert the system of discrete-time in the corresponding system of continuous-time using any transformation as discussed in the methodology. By using the bilinear transformation, i.e. *z=s+1*, the corresponding continuous-time transfer function of equation (5.37) is obtained from the discrete-time higher-order system (5.36) as follows:

Lower bound system

$$G_8(s) = \frac{1.648s^7+12.63s^6+40.96s^5+73.18s^4+77.5s^3+47.93s^2+15.56s+1.92}{\begin{array}{c}23.52s^8+186.4s^7+645.4s^6+1274s^5+1570s^4+1236s^3+608.4s^2 \\ +171.7s+21.44\end{array}} \quad (5.37a)$$

Upper bound system

$$G_8(s) = \frac{1.716s^7 + 13.15s^6 + 42.65s^5 + 76.25s^4 + 80.87s^3 + 50.2s^2 + 16.44s + 2.08}{24.48s^8 + 194.2s^7 + 672.8s^6 + 1330s^5 + 1640s^4 + 1293s^3 + 637.5s^2 + 180.3s + 22.56} \quad (5.37b)$$

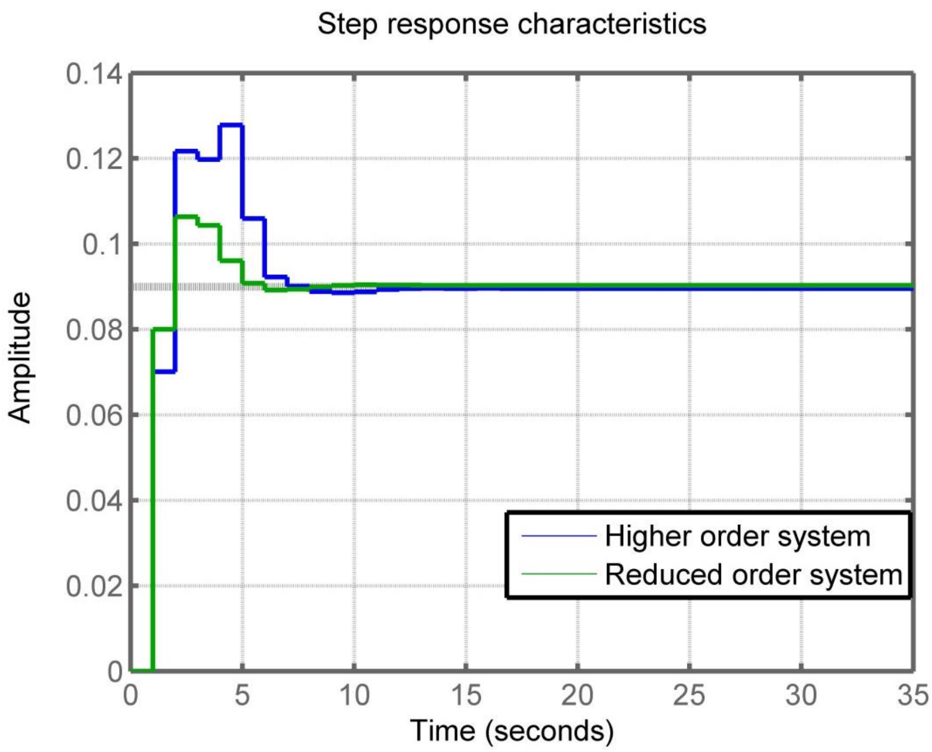
The aim is to reduce the 8th order system obtained in equation (5.37) in the 2nd order approximated system by following the implemented methodology. From partial fraction expansion of equation (5.37a), the eight poles of the lower bound discrete-order system are obtained as: $\sigma_{1,2}$= *-0.5311±0.2064i,* $\sigma_{3,4}$= *-0.639±0.3679i,* $\sigma_{5,6}$= *-1.0248±0.6969i,* $\sigma_{7,8}$= *-1.7677±0.4879i.* There will be two clusters in the lower bound reduced-order system as described in the methodology section that is to be formed from eight poles (four pair of complex conjugate poles) according to the order of MDIs. The cluster centres of both clusters are in the form of complex conjugate pair and are obtained as $\sigma_{c1,2}$= *-0.6465±0.3653i.* So, combining the two cluster centres and forming the 2nd order denominator equation described in equation (5.13) for the system of reduced-order. The denominator equation of reduced-order for lower bound is obtained as:

$$D_2(s) = s^2 + 1.293s + 0.5514 \quad (5.38)$$

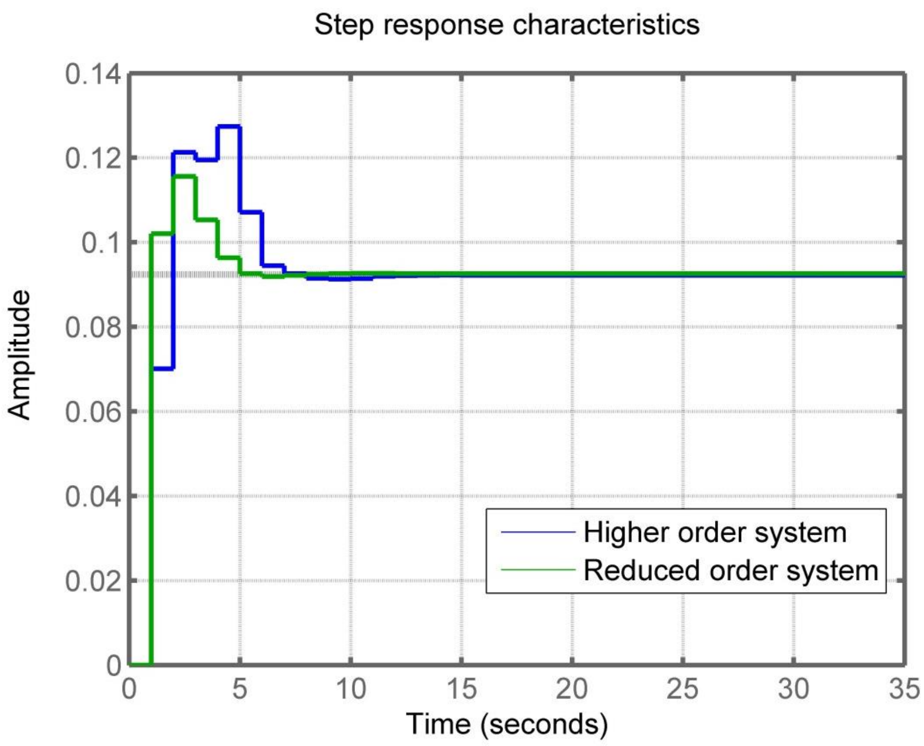
Similarly, for the upper bound system, eight poles are obtained from equation (5.37b) as $\sigma_{1,2}$= *-0.5456±0.2436i,* $\sigma_{3,4}$= *-0.6974±0.2665i,* $\sigma_{5,6}$= *-1.019±0.6580i,* $\sigma_{7,8}$= *-1.7038±0.425i* Now, the two clusters are formed from the complex conjugate pair of pole and hence the cluster centres are obtained as $\sigma_{c1,2}$= *-0.6865±0.2931i.* These cluster centres are combined after that to form the reduced system of 2nd order's denominator polynomial, shown as follows:

$$D_2(s) = s^2 + 1.373s + 0.5572 \quad (5.39)$$

Now, the system of reduced order's denominator equation, i.e. equation (5.38) and equation (5.39), is used for obtaining the system of reduced order's numerator. So, the next step is to find the value of unknown coefficients of the numerator by genetic algorithm (GA). The ISE in the step response of the system of higher-order and the approximated system of reduced-order is minimised by GA for upper bound as well as lower bound. After optimisation, the equation for the system of reduced-order is procured as follows:



(a)



(b)

Figure 5.34. System with discrete-time of higher and reduced order's step response characteristics for (a) lower bound (b) upper bound of example 3

$$G_2(s) = \frac{0.08s + 0.0498}{s^2 + 1.293s + 0.5514} \text{ (For lower bound)} \tag{5.40a}$$

$$G_2(s) = \frac{0.102s + 0.0526}{s^2 + 1.373s + 0.5572} \text{ (For upper bound)} \tag{5.40b}$$

Now from the inverse transformation, i.e. $s=z-1$, the continuous-time system is converted back to a discrete-time system. So after inverse transformation of the continuous-time system of equation (5.40a) and equation (5.40b), the 2nd order discrete-time system is obtained as follows:

$$G_2(z) = \frac{[0.08, 0.102] \cdot z + [-0.0302, -0.0494]}{[1,1] \cdot z^2 + [-0.707, -0.627] \cdot z + [0.2584, 0.1842]} \tag{5.41}$$

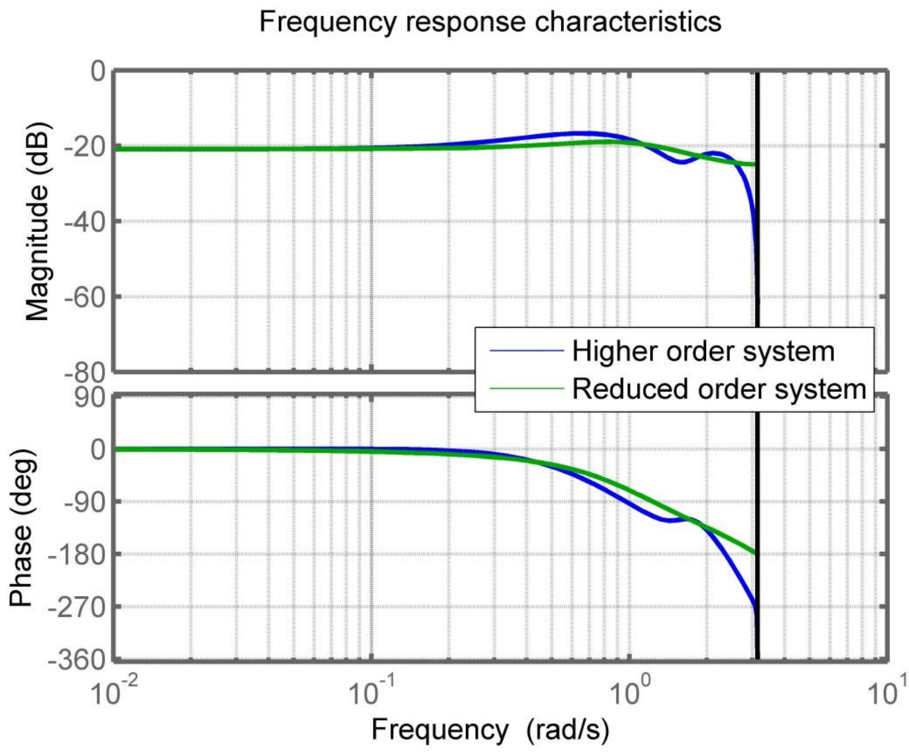
From the implemented technique, the reduced-order approximation of the higher-order system of equation (5.36) is obtained and shown in equation (5.41). The accuracy of the system of reduced-order in equation (5.41) is checked by its comparison with the higher-order system of equation (5.36) based on the frequency and step response behaviour of the system of higher and reduced-order. The step response behaviour for the lower bound system is depicted in figure 5.34.

Table 5.23. Value of errors and various parameters to compare lower bound (LB) and upper bound (UB) of higher system with discrete-time with 8th order and obtained system with discrete-time of 2nd order of example 3 for the unit step input

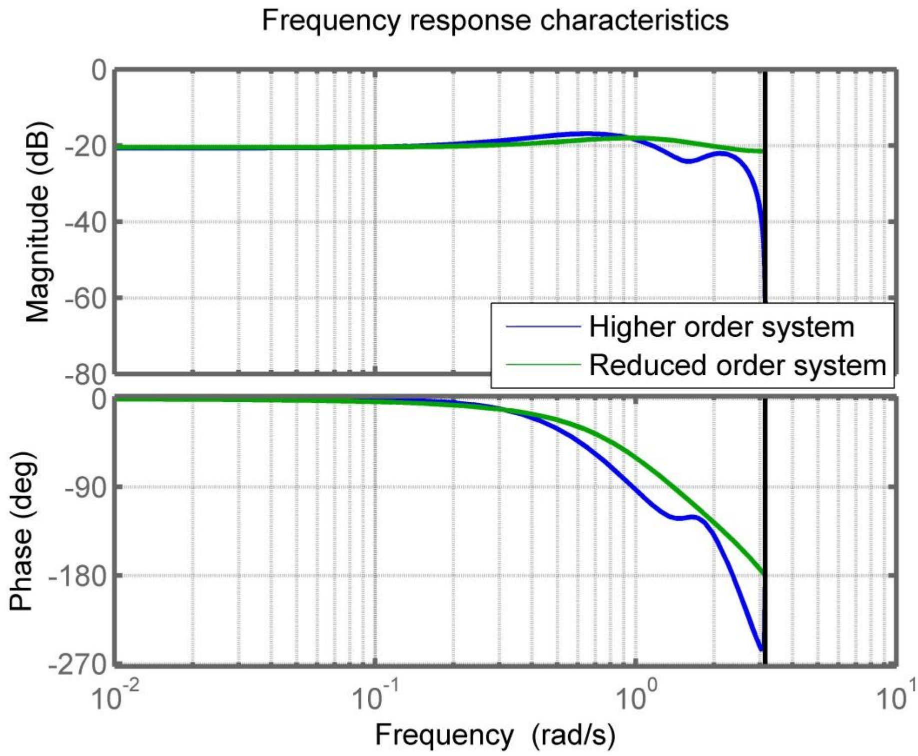
	Parameter	ISE	ITAE	IAE	OS (%)	RT (sec.)	ST (sec.)	SSE	GM	PM
LOWER BOUND	HOS	-	-	-	42.7	1	7	-	13.5	∞
	ROS	1.8×10^{-03}	0.787	0.114	33.7	1	5	0	17.8	∞
UPPER BOUND	HOS	-	-	-	38.1	1	7	-	13.8	∞
	ROS	2.4×10^{-03}	1.632	0.157	23.4	0	5	0	11.9	∞

The lower bound of the system of higher-order and corresponding system of reduced-order shows approximately the same behavioural characteristics in the whole time range. It is shown in figure 5.34(a). But, there is a little value of overshoot occurring in the response of the system of reduced-order. But, both system's rise time and value at steady-state are seen to be nearly equal from the step response behaviour. Similarly, figure 5.34(b) shows the upper bound of the system of higher-order and corresponding system of reduced-order follows approximately the same characteristics like settling and rise time. The overshoot exhibited by the system of reduced-order is a little higher than the system of higher-order. Moreover, the more accurate conclusion on the step response behaviour can be drawn from Table 5.23.

It is clear from Table 5.23 that for the lower bound system, the value of rise time for the system of higher and reduced-order is almost similar but having a slight deviation in settling time. Also, it is noted that the system of reduced order's overshoot is a little higher than the system of higher order's overshoot. Moreover, for input equal to the unit step, the error at steady state is zero in the lower bound reduced-order system obtained by implemented technique. Furthermore, while comparing the step response characteristics of the upper bound system, it is seen that rise time's value, settling time, and overshoot is almost similar for both of the systems. While analysing the error in the system of higher and reduced-order, it is seen that the value of ISE, IAE and ITAE for lower bound as well as upper bound system is shallow. It signifies that there is significantly less error amidst the system of higher and lower-order. So, the step response comparison in figure 5.34 and Table 5.23 depicts the time domain characteristics of the reduced system with discrete-time of 2nd order approximately matches with the higher 8th order discrete-time system. By analysing the system with discrete-time of higher and reduced order's frequency response behaviour in figure 5.35, it is found out that the system of higher and reduced order's lower bound and upper bound magnitude response is almost the same with corresponding higher-order systems over the complete frequency range. Furthermore, the phase response of the lower and upper bound of the system of reduced-order is also the same as the corresponding higher-order system. The main characteristic of frequency response, i.e., GM and PM, is found in Table 5.23.



(a)



(b)

Figure 5.35. System with discrete-time of higher and reduced order's frequency response characteristics for (a) lower bound (b) upper bound of example 3

The value of GM and PM of the lower and upper bound higher and reduced-order system is approximately the same. It is found that the frequency response of the system with discrete-time of higher order is almost similar to the reduced system with discrete-time.

The equivalency in the system of higher and reduced-order can also be proven by obtaining both systems' time moments. The similar value of the time moments of the two systems results that these systems are similar. Both systems first three time-moments are obtained from equation (4.17) by putting the value of $i=1, 2$ and 3. The first three time-moments of the lower bound higher-order system are $T_{ll1}= 0.0086$, $T_{ll2}= -0.7487$ and $T_{ll3}= 7.2404$ and for the upper bound higher-order system are $T_{lu1}= -0.0135$, $T_{lu2}= -0.6239$ and $T_{lu3}= 6.4651$. Similarly, for lower bound reduced-order system the first three time-moments are $T_{rl1}= -0.066$, $T_{rl2}= -0.6148$ and $T_{rl3}= 5.8297$ and of upper bound reduced-order system are $T_{ru1}= -0.0496$, $T_{ru2}= -0.246$ and $T_{ru3}= -5.2331$. It shows that the system of higher and reduced order's time moments obtained by implemented method is approximately the same. Hence, it results that the given 8th order system is similar to the derived 2nd order system.

Comparative investigations: For checking the superiority of the implemented technique over the other techniques studied in the literature, a comparative analysis is performed (Choudhary & Nagar, 2018; Glover, 1984; Moore, 1981). All methods are used to reduce the given 8th order system with discrete time, and a 2nd order reduced system with discrete time is obtained. The comparison of the lower bound system is conducted based on step response characteristics of the system of reduced-order procured through vivid methods, as depicted by figure 5.36. It is analysed from the figure that the response obtained by applying all techniques shows approximately similar responses except the technique presented by Chaudhary. The starting value and settling value of all responses are at the same amplitude except Chaudhary. The implemented technique is shown to possess a lower value of overshoot as compared to other techniques. Moreover, all responses except Chaudhary tend to show the exact value of error at steady-state by following a similar path as traversed by the original system. A more precise comparative analysis is performed by obtaining the characteristics of time-domain and error values and is depicted in Table 5.24.

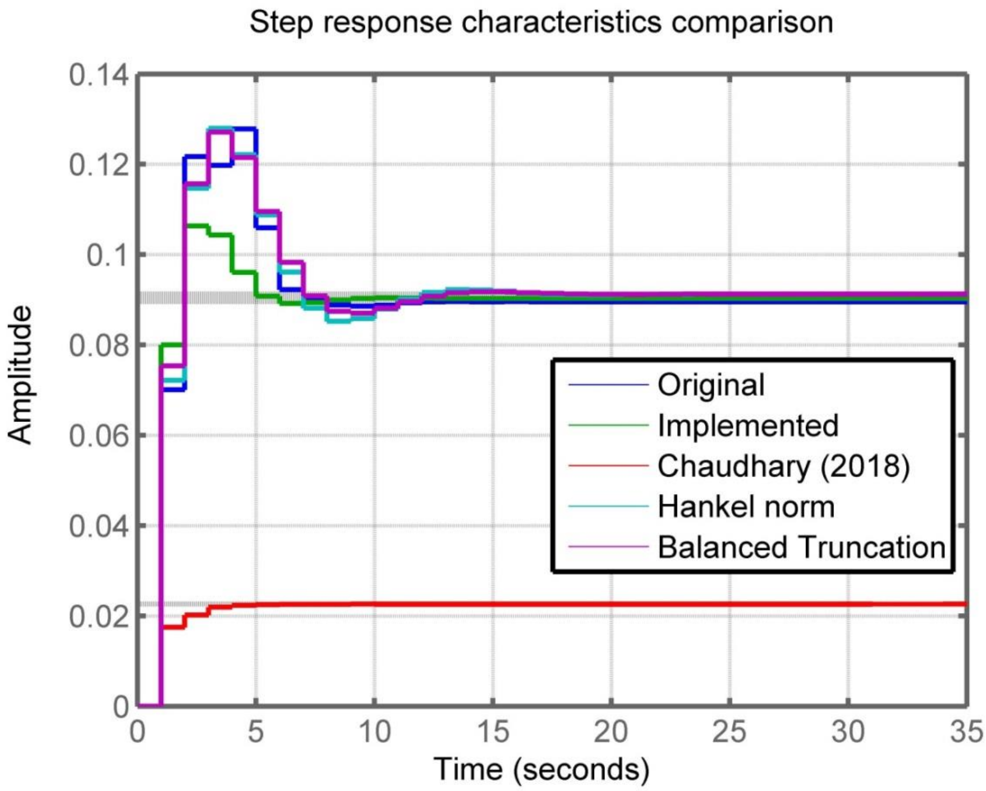


Figure 5.36. Comparative analysis of newly implemented technique with techniques studied for reducing the 8th order single variable discrete-time system based on step response characteristics for lower bound

It is clear from Table 5.24 that the rise time possessed by implemented technique, Hankel norm approximation and balanced truncation method is highly similar to the actual system. Moreover, the settling time of the implemented technique is highly similar to the actual system. Furthermore, the value of overshoot provided by implemented technique response is lower than the original system. It is observed that the most similar value of overshoot is obtained for the Hankel norm and balanced truncation technique. Moreover, the value of steady-state error is zero for implemented system response, while other techniques are providing some amount of steady-state error. Hence it is advised that the implemented technique provides better values of time-domain performance characteristics. While analysing the error value (ISE, ITAE, IAE), it has resulted that the value of ISE possessed by implemented technique is lower than all the techniques used in the comparison. The implemented technique shows the most negligible value of ISE, ITAE and IAE between all techniques discussed in comparative analysis. Hence, the overall comparison shows that the implemented technique possesses the better value of all time-domain characteristics and error parameters discussed in the comparative analysis for the lower bound of the original system. So from the comparative analysis, it is revealed that the technique presented in the research shows better performance compared to the other techniques used in the comparative analysis as observed in figure 5.36 and Table 5.24 for the lower bound of the original system given by equation (5.36).

Table 5.24. Comparative analysis of errors and time-domain performance parameters for lower bound of the discrete-time system of example 3 subjected to the unit step input

Parameter	ISE	ITAE	IAE	OS (%)	RT (sec.)	ST (sec.)	SSE
Original	-	-	-	42.71	1	7	-
Implemented	0.0018	0.787	0.1140	33.76	1	5	0
Choudhary (2018)	0.1754	42.57	2.4508	0	2	4	0.06
Hankel norm	0.0046	1.165	0.1900	40.21	1	11	0.01
Balanced Truncation	0.0046	1.123	0.1900	39.31	1	11	0.01

Similarly, the comparison of the upper bound system is conducted based on step response characteristics of the system of reduced-order procured through vivid methods, as depicted in figure 5.37. It is analysed from the figure that the response obtained by applying all techniques shows approximately similar responses except the technique presented by Chaudhary. The starting value and settling value of all responses are at the same amplitude except Chaudhary. Moreover, it is analysed that the implemented technique possesses a lower value of overshoot than other techniques. Furthermore, all responses except that of Chaudhary tends to show the exact figure of error at steady-state by traversing a pretty similar path of the original system. A more precise comparison is done by obtaining time-domain characteristics and error values, as shown in Table 5.25. It shows that the rise time possessed by the hankel norm approximation and balanced truncation technique is closer to the original system and that possessed by implemented. Chaudhary technique is approximately similar to the initial system. Moreover, the settling time possessed by implemented technique is highly the same as the original system compared to various methods.

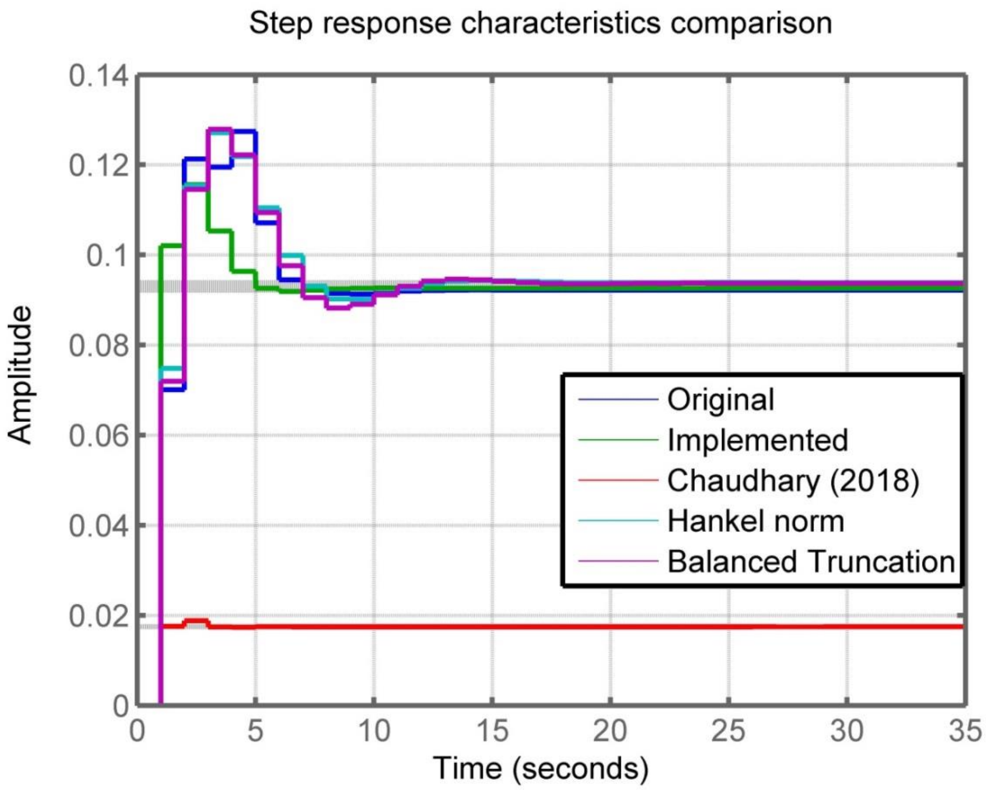


Figure 5.37. Comparative analysis of newly implemented technique with techniques studied for reducing 8th order single variable discrete-time system based on step response characteristics for upper bound

Furthermore, the value of overshoot provided by implemented technique response is lower than the original system, making the system less oscillatory. It is observed that the most similar value of overshoot is obtained for the Hankel norm and balanced truncation technique. Moreover, the value of steady-state error is zero for implemented system response, while other techniques are providing some amount of steady-state error. Hence it is advised that the implemented technique provides better values of time-domain performance characteristics. While analysing the error value (ISE, ITAE, IAE), it has resulted that the value of ISE possessed by implemented technique is lower than all the techniques used in the comparison. The implemented technique shows the most negligible value of ISE, ITAE and IAE between all techniques discussed in comparative analysis. Hence, the overall comparison shows that the implemented technique possesses the better value of all time-domain characteristics and error parameters discussed in the comparative investigation for an upper bound of the original system. So from the comparative analysis, it is revealed that the technique presented in the research shows better performance compared to the other techniques used in the comparative analysis as observed in figure 5.37 and Table 5.25 for the upper bound of the original system given by equation (5.36).

Table 5.25. Comparative analysis of errors and time-domain performance parameters for upper bound of the discrete-time system of example 3 subjected to the unit step input

Parameter	ISE	ITAE	IAE	OS (%)	RT (sec.)	ST (sec.)	SSE
Original	-	-	-	38.15	1	7	-
Implemented	0.0024	1.6320	0.1579	23.46	0	5	0
Choudhary (2018)	0.2113	47.394	2.698	7.73	0	3	0.08
Hankel norm	0.1153	16.136	0.3288	35.40	1	11	0
Balanced Truncation	6.8627	10.152	0.2236	36.44	1	11	0.001

From the reduction of the 8th order discrete-time system to procure a system of 2nd order by the method implemented in the research work, it is clear that the implemented method provides a fair approximation of the system of higher-order with similar time and frequency domain characteristics and equivalent time moments. Also, the implemented technique provides a stable reduced-order system. Furthermore, the technique implemented here also provides better time-domain characteristics for step input than other techniques presented in the literature.

So, based on the examples of discrete-time systems presented in this section, it is clear that the implemented technique is also suitable for these systems along with continuous-time systems. Hence, it is clear that the implemented technique applies to all types of systems.

5.3 REDUCTION OF MULTIVARIABLE (MIMO) SYSTEMS

It is analysed in previous sections that the implemented technique provides a good approximation of the continuous and discrete-time systems. But, the implemented technique is tested on the single variable systems only. A MOR technique can't be judged based on only single variable systems in this technologically advanced era. So, it is also the demand of industry that the MOR technique must reduce multi variable systems. Hence, the multi variable system's reduction is performed to analyse the ability of implemented technique over multivariable systems of up to 19 orders.

Example 1: Consider a multivariable system of 6th order as represented by equation (5.42) in the transfer function form. It is clear from the equation that two inputs are applied to the given system, and two outputs are obtained.

$$G_6(s) = \begin{bmatrix} \dfrac{2(s+5)}{(s+1)(s+10)} & \dfrac{(s+4)}{(s+2)(s+5)} \\ \dfrac{(s+10)}{(s+1)(s+20)} & \dfrac{(s+6)}{(s+2)(s+3)} \end{bmatrix} \tag{5.42}$$

The generalised form of it is represented in equation (5.43) as follows:

$$G(s) = \frac{1}{D(s)}\begin{bmatrix} a_{11}(s) & a_{12}(s) \\ a_{21}(s) & a_{22}(s) \end{bmatrix} \tag{5.43}$$

The multinomial of the denominator for all the four subsystems are the same as given by *'D(s)'*, and the numerator polynomials are different and presented by *'a_{ij}'*, having *i=j=1,2.* The equation for denominator and numerator is calculated as

$$D(s) = (s+1)(s+2)(s+3)(s+5)(s+10)(s+20)$$
$$= s^6 + 41s^5 + 571s^4 + 3491s^3 + 10060s^2 + 13100s + 6000$$
$$a_{11}(s) = 2s^5 + 70s^4 + 762s^3 + 3610s^2 + 7700s + 6000$$
$$a_{12}(s) = s^5 + 38s^4 + 459s^3 + 2182s^2 + 4160s + 2400$$
$$a_{21}(s) = s^5 + 30s^4 + 331s^3 + 1650s^2 + 3700s + 3000$$
$$a_{22}(s) = s^5 + 42s^4 + 601s^3 + 3660s^2 + 9100s + 6000$$

For deriving the coefficients of numerator and denominator of the system of second-order, firstly, the six poles are fetched from the denominator multinomial and written as $\sigma_1= -1$, $\sigma_2= -2$, $\sigma_3= -3$, $\sigma_4= -5$, $\sigma_5= -10$ and $\sigma_6= -20$. Then from methodology proposed from the IPCG technique, the MDIs (η_{11}, η_{12}, η_{21}, η_{22}) are obtained from partial fraction as given in Table 5.26 and helps in finding the cluster centre of the two clusters that are formed from the six poles of the higher-order system.

Table 5.26. Value of MDI for the four subsystems of example 1

	η_{11}	η_{12}	η_{21}	η_{22}
σ_1=-1	0.8889	0	0.4737	0
σ_2=-2	0	0.3333	0	2
σ_3=-3	0	0	0	1
σ_4=-5	0	0.0667	0	0
σ_5=-10	0.1111	0	0	0
σ_6=-20	0	0	0.0263	0

From Table 5.26, the MDI associated with pole σ_1=-1 is highest amongst other poles. So the first cluster is formed of the first pole, and all other poles are lying in the second cluster having σ_2=-2 with the second highest MDI value. After solving the equation (4.9), the cluster centres are obtained as $\sigma_{c1}= -1$, $\sigma_{c2}= -2.3844$. These cluster centres are the poles of the system of reduced-order. Hence the equation of denominator multinomial of the system of reduced-order is obtained as follows:

$$D_2(s) = s^2 + 3.068s + 2.068$$

A genetic algorithm is applied by optimizing the ISE to obtain the multinomial numerator for the four subsystems. By optimisation, the system of reduced-order is obtained, as shown in equation (5.44).

$$G_2(s) = \frac{1}{D_r(s)}\begin{bmatrix} c_{11}(s) & c_{12}(s) \\ c_{21}(s) & c_{22}(s) \end{bmatrix} = \frac{\begin{bmatrix} 1.22s + 2.068 & 0.8608s + 0.8271 \\ 0.5538s + 1.034 & 1.6135s + 2.068 \end{bmatrix}}{s^2 + 3.068s + 2.068} \quad (5.44)$$

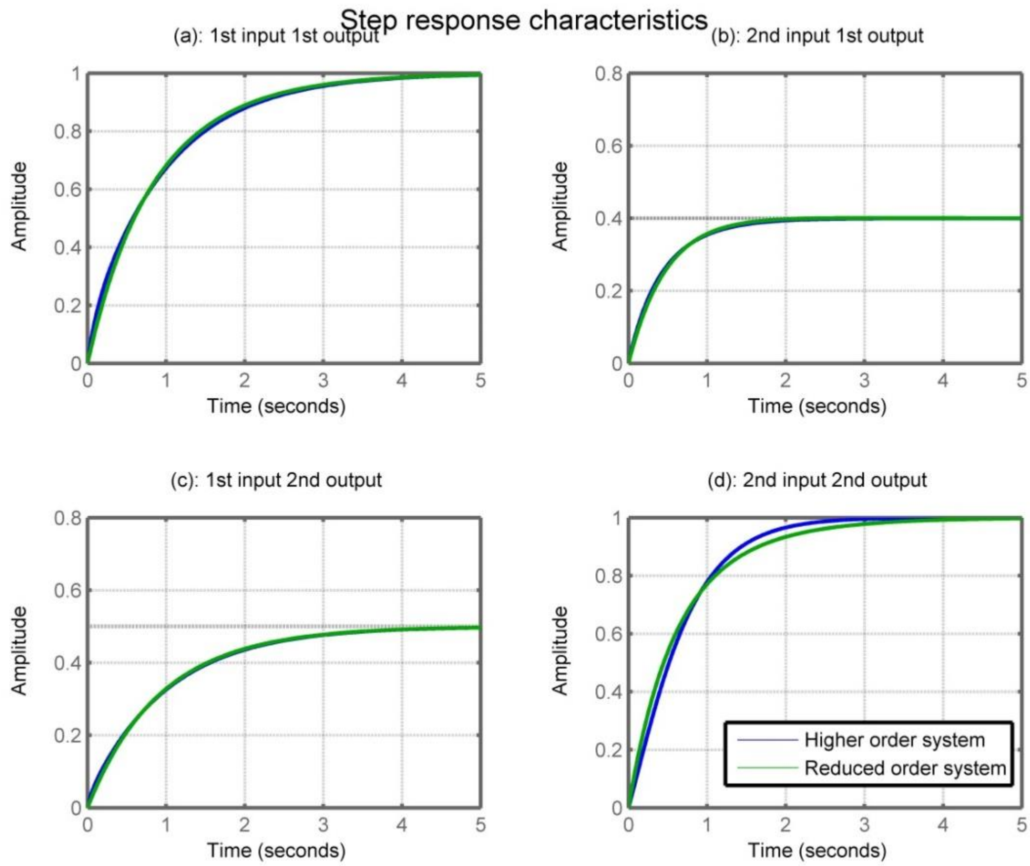


Figure 5.38. Step response characteristics of the multivariable system of high and low order obtained from implemented IPCG technique on example 1

So from the implemented technique, the approximated system of reduced-order is procured in equation (5.44). The accuracy of the system of reduced-order is checked by comparing with the system of the higher-order system based on the behaviour of step and step frequency response of both systems. The step response behaviour for the complete system is revealed in figure 5.38, which depicts that the response of the reduced-order of 1st input 1st output combination is exactly overlapping the response of the related system of higher-order. Similarly, the response of the system of reduced-order of 1st input 2nd output and 2nd input 1st output is overlapping the response of the corresponding higher-order system as revealed in figure 5.38 (b) and 5.38 (c). From figure 5.38 (d), it is clear that there is a minor variation in response of the reduced-order system before reaching the steady-state. But after reaching the steady-state, the response of the higher and reduced system of the 2nd input 2nd output combination becomes similar. A more accurate conclusion on the step response behaviour can be drawn from Table 5.27. It is clear from the table that for in the :(1) and out:(1) subsystem, the value of rising time and time of settling down for the system of high and low order is almost similar. The value of steady-state error is observed as zero with no overshoot in the reduced system with zero overshoot in the higher-order system. Also, it is noted that the value of ISE is shallow, resulting in an accurate approximation of the system of higher-order. Moreover, the value of ITAE and IAE is also shallow. So, the reduced-order approximation is said to be the replica of a higher-order system. Furthermore, for the in:(2) and out:(1) subsystem, it is seen that there is a minute variation in rising time and time to settle down for the system of reduced-order procured from the implemented technique. But, the value of overshoot is precisely matching the overshoot of the higher-order system with zero steady-state error in the system of reduced and higher-order. Moreover, the value of ISE is also shallow, promising high accuracy in the system of lower and higher-order. Also, the low value of ITAE and IAE signifies the high accuracy of the approximated system of reduced-order.

Similarly, for the subsystem in:(1) and out: (2), the value of rising time and time of settling down of higher and reduced-order systems is almost similar to zero error at the final time and a similar value overshoot. Hence, the time-domain characteristics of both systems are almost similar, and the steady-state value matches precisely. Also, the low value of ISE error is obtained after reducing the 6^{th} order system by implemented technique. Hence, the low error signifies the high accuracy of the obtained reduced system. The low value of other others ITAE and IAE are also obtained in the reduced-order approximation fetched by implemented technique. Furthermore, the in:(2) and out:(2) subsystems, are having the rise time of the higher and reduced system are almost similar with a bit of variation in time of settling both systems. But the overshoot is similar for both systems with the same value at the finalised stage, with zero value of stabilized state error for both systems. Moreover, ISE is reduced to a great extent; hence the difference between the two systems is reduced by implemented technique. The value of ITAE and IAE errors is also low, hence providing an accurate reduced-order system. So, the step response comparison in figure 5.38 and Table 5.27 shows that the characteristics of the time-domain for the multivariable system of 2nd order approximately match with the higher 6th order system.

Table 5.27. Value of errors and various parameters to compare 6th and 2nd order reduced multivariable systems of example 1 for a unit step input

	Parameter	ISE	ITAE	IAE	OS (%)	RT (sec.)	ST (sec.)	SSE	GM	PM
Input 1, Output 1	**HOS**	-	-	-	0	2.12	3.79	-	∞	180
	ROS	7.602×10^{-04}	0.053	0.040	0	2.01	3.68	0	∞	180
Input 2, Output 1	**HOS**	-	-	-	0	1.02	1.86	-	∞	∞
	ROS	6.93×10^{-05}	0.018	0.013	0	0.98	1.65	0	∞	∞
Input 1, Output 2	**HOS**	-	-	-	0	2.18	3.85	-	∞	∞
	ROS	7.034×10^{-05}	0.013	0.011	0	2.11	3.80	0	∞	∞
Input 2, Output 2	**HOS**	-	-	-	0	1.33	2.27	-	∞	∞
	ROS	3.5×10^{-03}	0.176	0.105	0	1.59	3.10	0	∞	∞

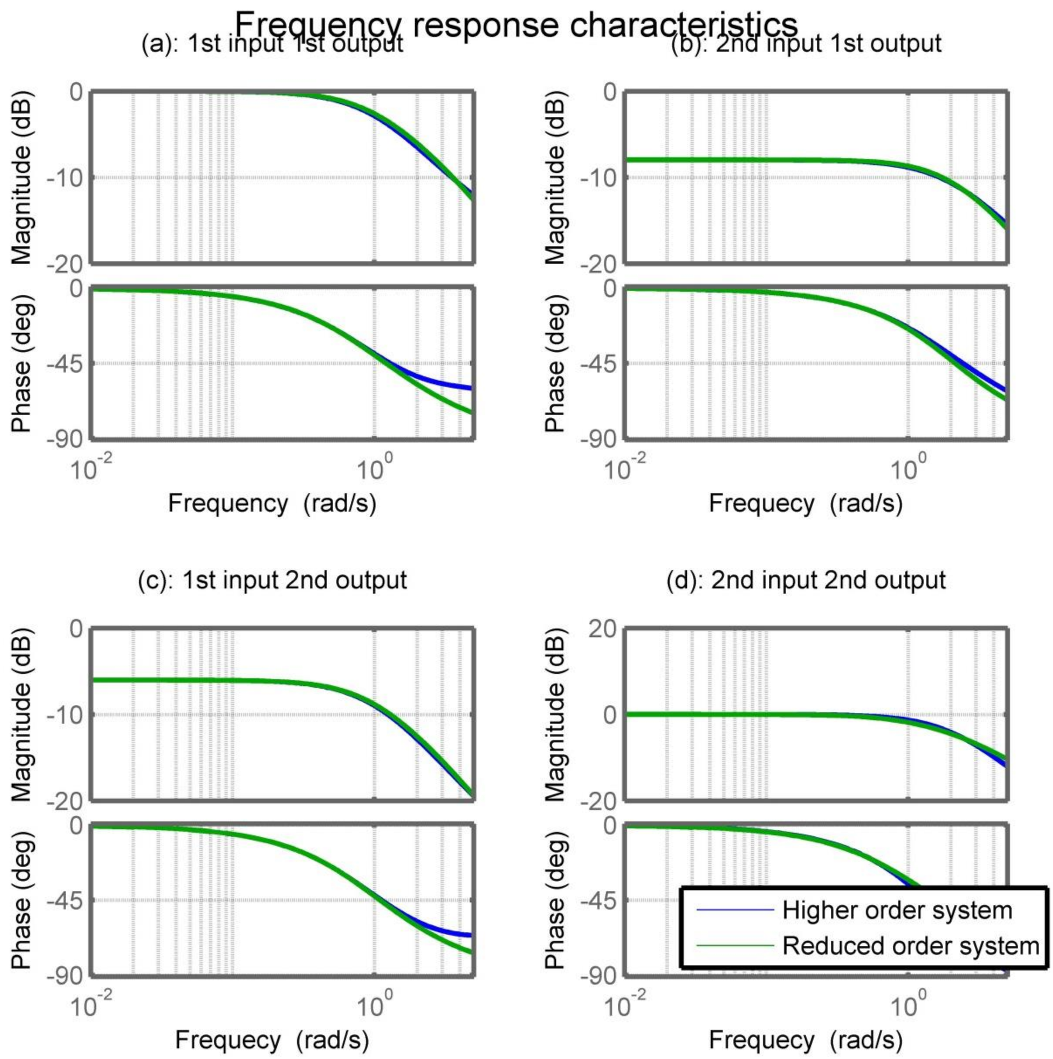


Figure 5.39. Frequency response characteristics of the multivariable system of higher and reduced-order obtained by IPCG technique on example 1

Analysing the frequency-based behaviour of the multivariable system of higher and reduced-order, figure 5.39 depicts that the gain response of the reduced-order systems is almost the same with corresponding higher-order systems in the complete frequency range for all subsystems. Furthermore, the phase response of all reduced-order subsystems is also found to be similar to the corresponding higher-order system at low frequencies. At the same time, there is a slight deviation in phase response at the high-frequency range. The main characteristic of frequency response, i.e., GM and PM, is noted from Table 5.27, which results that the value of GM and PM of the higher and reduced-order system is the same for all subsystems. So, it has resulted from the frequency-based characteristics that the higher-order system's response is almost identical to that of the system of reduced-order procured by the IPCG algorithm.

The equivalency in the system of higher and reduced-order is also proved by fetching the time moments. The similar value of the time moments of the two systems results that these systems are similar. The first three time-moments are procured for both systems from equation (4.17) by putting the value of *i=1*, *2* and *3*. The first three-time moments of the system of higher and reduced-order are obtained in Table 5.28. It shows that the higher-order system's time moments is approximately the same as the system of reduced-order. Hence, it results that the given 8th order system is similar to the derived 2nd order system.

Table 5.28. First three-time moments of higher and reduced multivariable system of example 1

Time moments	Higher-order system				Reduced-order system			
	In:(1), Out:(1)	In:(2), Out:(1)	In:(1), Out:(2)	In:(2), Out:(2)	In:(1), Out:(1)	In:(2), Out:(1)	In:(1), Out:(2)	In:(2), Out:(2)
First moment	-0.9	-0.18	-0.475	-0.666	-0.893	-0.177	-0.474	-0.704
Second moment	1.78	0.172	0.947	0.777	1.684	0.138	0.922	1.121
Third moment	-5.334	-0.253	-2.842	-1.277	-4.903	-0.103	-2.731	-2.948

Comparative investigation: For checking the superiority of the implemented technique over other techniques, the comparison of the implemented technique with other techniques (Narwal & Prasad, 2017; Parmar et al., 2007a; Prajapati & Prasad, 2020b; Sikander & Prasad, 2015b, 2017; Tiwari & Kaur, 2020a; Vishwakarma & Prasad, 2009) is performed. All techniques are used to reduce the given 6th order multivariable system, and a system of 2nd order is obtained. The comparison is conducted based on the step input based characteristics of the system of reduced-order procured by different techniques, as depicted in figure 5.40. The performance characteristics in time-domain and ISE, ITAE and IAE, are also utilised for the comparative analysis as shown in Table 5.29.

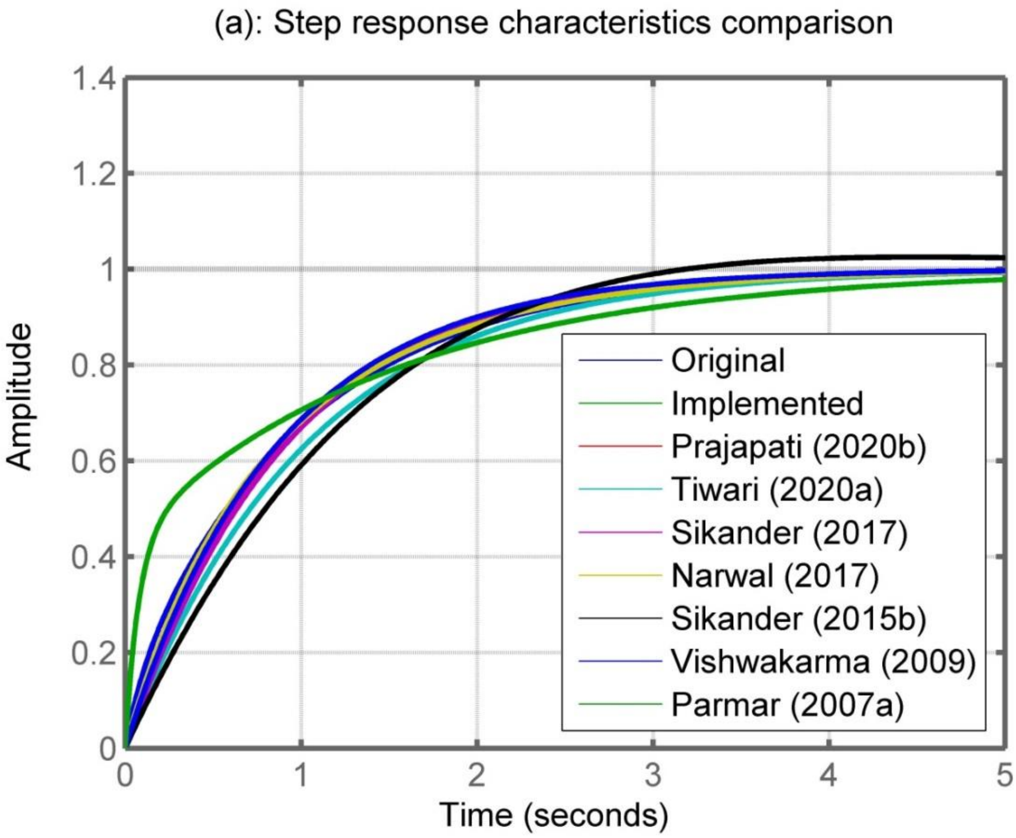


Figure 5.40 (a). Comparative investigation of IPCG technique with literature techniques for the reduction of 6th order multivariable system based on step response characteristics for 1st input, 1st output combination

It is analysed from figure 5.40 (a) that the step response of in:(1) out:(1) subsystem is obtained by applying all techniques shows approximately similar response except the technique of Sikander (2015b) and Parmar. The starting value of all responses is at the same amplitude. There is a slight difference in the various step responses, as is clear from the figure. Most of the responses are emerging from the same point and terminating at the same point. The more precise comparative analysis for in:(1) out:(1) subsystem is performed by obtaining the value of time-domain characteristics and error values and is revealed in Table 5.29 (a). It clears that the rise time of all subsystems of reduced-order is the same as that of the subsystem of higher-order, i.e. in:(1) out:(1) except the response of Sikander (2015b), Vishwakarma and Parmar.

Table 5.29 (a). Comparative analysis of errors and time-domain performance parameters for 1st input, 1st output combination of 6th order multivariable system of example 1 subjected to the unit step input

Parameter	ISE $\times 10^{-04}$	ITAE	IAE	OS (%)	RT (sec.)	ST (sec.)	SSE
Original	-	-	-	0	2.12	3.79	-
Implemented	7.602	0.053	0.040	0	2.01	3.68	0
Prajapati (2020b)	9.005	0.060	0.043	0	2.01	3.66	0
Tiwari (2020a)	58	0.134	0.116	0	2.22	3.94	0
Sikander (2017)	20	0.123	0.072	0	1.95	3.46	0
Narwal (2017)	3.942	0.514	0.036	0	2.05	3.74	0.001
Sikander (2015b)	167	0.768	0.283	2.5	2.01	5.56	0
Vishwakarma (2009)	15	0.147	0.074	0	1.90	3.40	0
Parmar (2007a)	227	0.551	0.259	0	2.63	5.13	0

Similarly, the settling time of implemented technique and other techniques is similar to the original system except for Sikander (2015b), Vishwakarma and Parmar. The overshoot possessed by Sikander (2015) is different from that of the higher-order system, while all other techniques possess the exact value of overshoot. Moreover, the error of finalised state is zero for all the subsystems except that of Narwal. From the analysis of errors, it is clear that the value of ISE is lowest for Narwal and second-lowest for implemented technique, while ITAE and IAE is lowest for Narwal's technique and implemented technique. So, the comparative study results that the reduction technique presented by Narwal provides approximately the same response as that of implemented technique. These two techniques provide better results compared to other techniques.

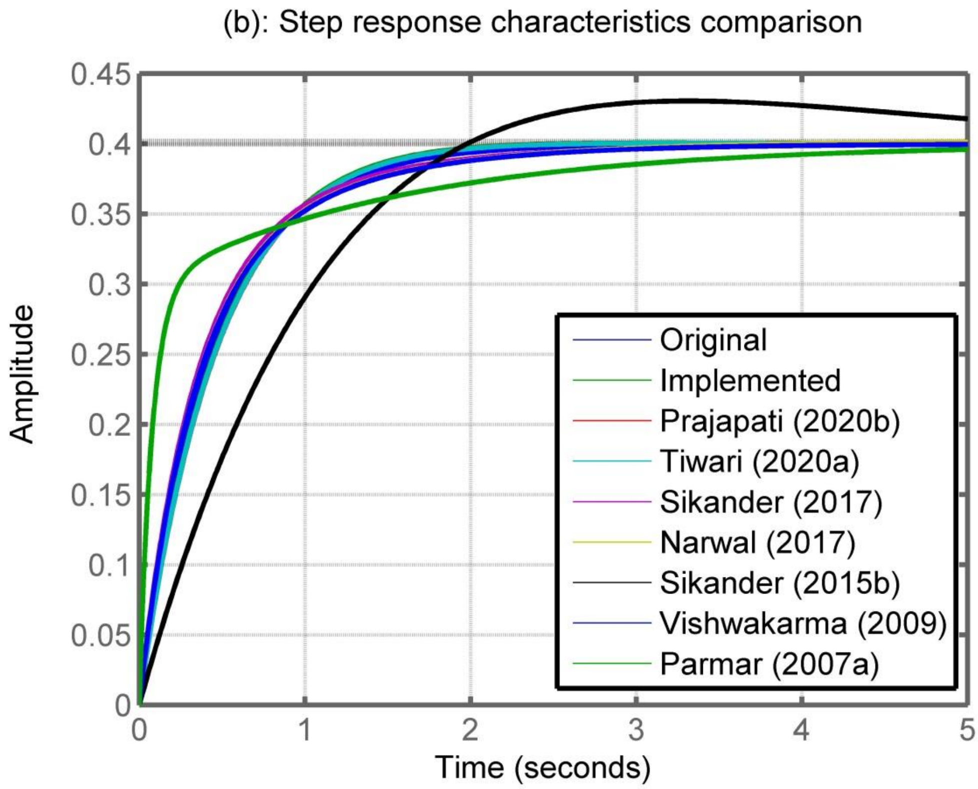


Figure 5.40 (b). Comparative analysis of newly implemented technique with techniques given in the literature for the reduction of 6th order multivariable system based on step response characteristics for 2nd input, 1st output combination

It is analysed from figure 5.40 (b) that the step response of in:(2) out:(1) subsystem is obtained by applying all techniques shows approximately similar response except the technique of Sikander (2015b) and Parmar. The starting value of all responses is at the same amplitude. There is a slight difference in the various step responses, as is clear from the figure. Most of the responses are emerging from the same point and terminating at the same point. The more precise comparative analysis for in:(2) out:(1) subsystem is performed by obtaining the value of time-domain characteristics and error values and is revealed in Table 5.29 (b). Table clears that the value of rising time of all subsystems of reduced-order is the same as that of the subsystem of the higher-order system, i.e. in:(2) out:(1) except the response of Sikander (2015b) and Parmar.

Similarly, the settling time of the implemented technique and Tiwari's techniques is similar to the original system, while all other techniques have a considerable change in settling time. The techniques presented by Prajapati, Tiwari and Sikander (2015) possess some overshoot that is different from that of the higher-order system. All other techniques possess the exact value of overshoot as that of the original system. Moreover, the error at the finalised stage is zero for all the systems except that of Narwal and Sikander (2015). From the analysis of errors, it is clear that the value of ISE is the lowest for implemented technique. Also, the value of ITAE and IAE is the lowest for implemented technique. So, the comparative study results that the reduction technique presented by implemented technique and these two techniques provides better results than other techniques.

Table 5.29 (b). Comparative analysis of errors and time-domain performance parameters for 2nd input, 1st output combination of 6th order multivariable system of example 1 subjected to the unit step input

Parameter	ISE $\times 10^{-05}$	ITAE	IAE	OS (%)	RT (sec.)	ST (sec.)	SSE
Original	-	-	-	0	1.02	1.86	-
Implemented	6.93	0.018	0.013	0	0.98	1.65	0
Prajapati (2020b)	8.01	0.019	0.013	0.14	0.99	1.68	0
Tiwari (2020a)	7.23	0.016	0.012	0.12	0.99	1.69	0
Sikander (2017)	10.96	0.026	0.017	0	1.01	2.21	0
Narwal (2017)	12.28	0.372	0.042	0	1.08	2.48	0.0019
Sikander (2015b)	960	0.505	0.202	7.66	1.39	6.27	0.0001
Vishwakarma (2009)	7.84	0.031	0.016	0	1.06	2.33	0
Parmar (2007a)	690	0.191	0.119	0	1.43	3.91	0

It is analysed from figure 5.40 (c) that the step response of in:(1) out:(2) subsystem is obtained by applying all techniques shows approximately similar response except the technique of Sikander (2015b) and Parmar. The starting value of all responses is at the same amplitude. Most of the responses are emerging from the same point and terminating at the same point. The more precise comparative analysis of the subsystem is performed by obtaining time-domain characteristics and error values and is depicted in Table 5.29 (c).

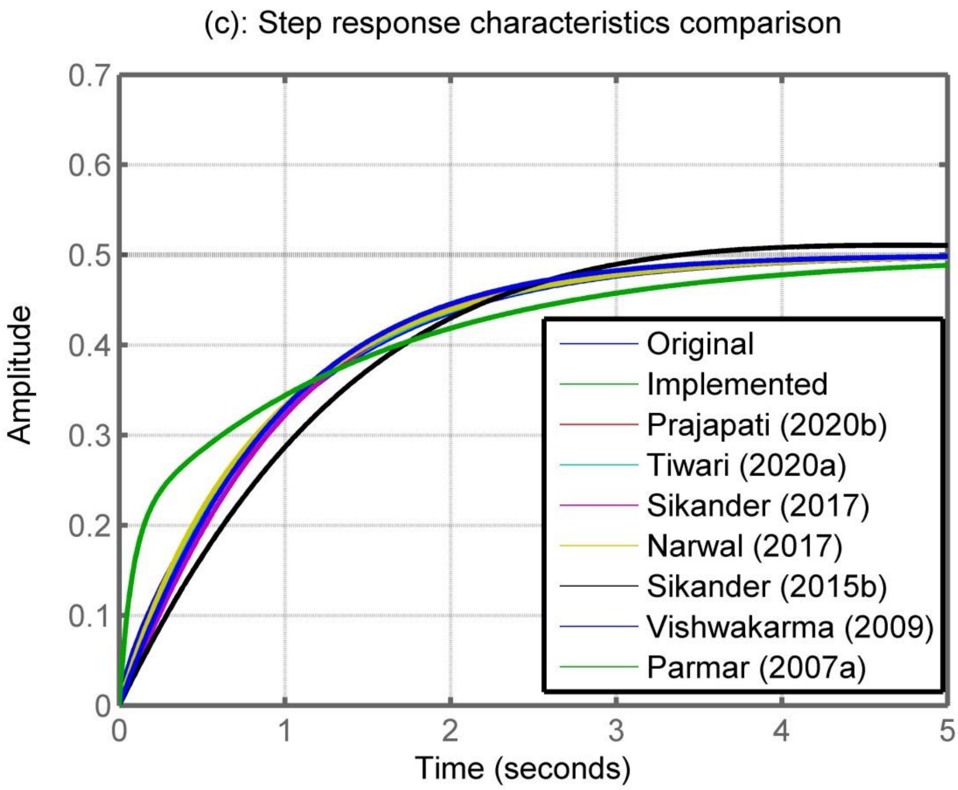


Figure 5.40 (c). Comparative analysis of newly implemented technique with techniques given in the literature for the reduction of 6th order multivariable system based on step response characteristics for 1st input, 2nd output combination

Table 5.29 (c) depicts that the rising time of all subsystems of reduced-order is approximately the same as that of the subsystem of higher-order except the response of Sikander (2017), Vishwakarma and Parmar. Similarly, the settling time of implemented technique and Tiwari's techniques is obtained as most similar to the original system, while all other techniques have a considerable change in settling time. The techniques presented by Sikander (2015b) possess some overshoot which is different from that of the higher-order system, while all other techniques possess the same value of overshoot as that of the original system. Moreover, the error at the final state is zero for all the systems except that of Narwal and Sikander (2015b), which is not desirable. From the analysis of errors, it is clear that the value of ISE is the lowest for implemented technique. Also, the value of ITAE and IAE is the lowest for implemented technique. So, the comparative study results that the reduction technique presented by implemented technique and these two techniques provides better results than the other techniques.

Table 5.29 (c). Comparative analysis of errors and time-domain performance parameters for 1st input, 2nd output combination of 6th order multivariable system of example 1 subjected to the unit step input

Parameter	ISE $\times 10^{-05}$	ITAE	IAE	OS (%)	RT (sec.)	ST (sec.)	SSE
Original	-	-	-	0	2.18	3.85	-
Implemented	7.034	0.013	0.011	0	2.11	3.80	0
Prajapati (2020b)	8.969	0.019	0.013	0	2.10	3.77	0
Tiwari (2020a)	7.067	0.013	0.011	0	2.11	3.81	0
Sikander (2017)	42.67	0.060	0.034	0	2.01	3.53	0
Narwal (2017)	8.255	0.034	0.015	0	2.10	3.79	0.0001
Sikander (2015b)	310	0.359	0.126	2.17	2.10	5.29	0.0001
Vishwakarma (2009)	29.98	0.071	0.034	0	1.97	3.48	0
Parmar (2007a)	620	0.290	0.136	0	2.72	5.22	0

It is analysed from figure 5.40 (d) that the step response of in:(2) out:(2) subsystem is obtained by applying all techniques shows approximately similar response except the technique of Sikander (2015b) and Parmar. The starting value of all responses is at the same amplitude. Most of the responses are emerging from the same point and terminating at the same point. The more precise comparative analysis of the subsystem is performed by obtaining the value of time-domain characteristics and error values presented in Table 5.29 (d). It clears that the value of rising time of all subsystems of reduced-order is approximately the same as that of the subsystems of higher-order except the response of Narwal, Sikander (2015b), Vishwakarma and Parmar.

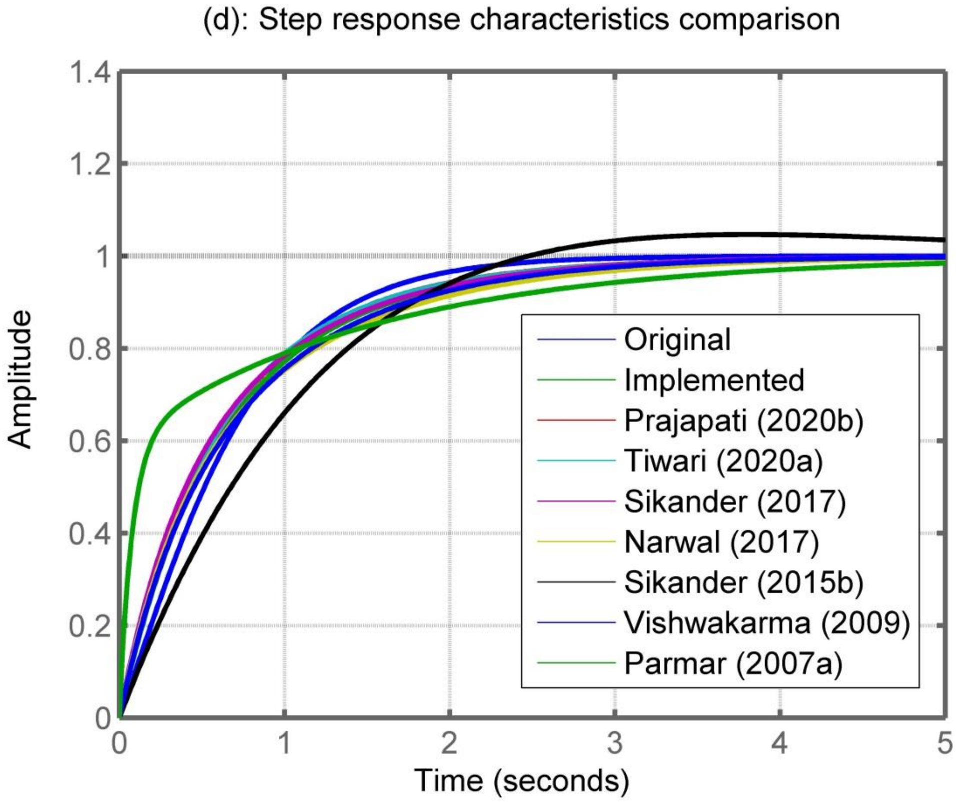


Figure 5.40 (d). Comparative analysis of newly implemented technique with techniques given in the literature for the reduction of 6th order multivariable system based on step response characteristics for 2nd input, 2nd output combination

Similarly, the settling time of Narwal, Sikander (2015b) and Parmar is deviated from the original system, while all other techniques have an approximately similar value of settling time. The techniques presented by Sikander (2015b) possess some overshoot which is different from that of the higher-order system, while all other techniques possess the exact value of overshoot as that of the original system. Moreover, the error at the final stage is zero for all the systems except that of Narwal, which is not desirable. From the analysis of errors, it is clear that the value of ISE is the lowest for implemented technique. Also, the value of ITAE and IAE is the lowest for implemented technique. So, the comparative study results that the reduction technique presented by implemented technique and these two techniques provides better results than the other techniques.

The reduction of a MIMO system of 6^{th} order to procure a system of 2^{nd} order by the IPCG method implemented in the research work is done. The implemented method provides a more accurate approximation of the system of higher-order with approximately similar time and frequency domain characteristics and equivalent time moments. Also, the implemented technique provides a stable reduced-order system. Furthermore, the technique implemented here also provides better time-domain characteristics for step input than other techniques presented in the literature.

Table 5.29 (d). Comparative analysis of errors and time-domain performance parameters for 2^{nd} input, 2^{nd} output combination of 6^{th} order multivariable system of example 1 subjected to the unit step input

Parameter	ISE $\times 10^{-05}$	ITAE	IAE	OS (%)	RT (sec.)	ST (sec.)	SSE
Original	-	-	-	0	1.33	2.27	-
Implemented	3.5	0.176	0.105	0	1.59	3.10	0
Prajapati (2020b)	3.8	0.153	0.009	0	1.49	2.91	0
Tiwari (2020a)	4.0	0.161	0.103	0	1.49	2.95	0
Sikander (2017)	6.1	0.180	0.123	0	1.57	3.02	0
Narwal (2017)	7.0	0.335	0.162	0	1.79	3.44	0.0002
Sikander (2015)	20.0	0.886	0.315	4.65	1.67	6.04	0
Vishwakarma (2009)	4.7	0.220	0.125	0	1.68	3.15	0
Parmar (2007)	68.3	0.713	0.410	0	2.13	4.61	0

Example 2: The transfer function of a 7th order power system of the thermal-thermal non-reheated type consisting of two areas is presented by equation (5.45). The 7th order system is to be reduced to a system of 3rd order by the IPCG technique. A 2x2 MIMO system having two inputs and two outputs is given. It is also clear from the equation (5.45) that the system is symmetrical about its input and output combinations. The numerator equation of the four subsystems is given by a_{11}, a_{12}, a_{21} and a_{22}, and the denominator equation is presented by *D(s).*

$$G_7(s) = \frac{1}{D_7(s)} \begin{bmatrix} a_{11}(s) & a_{12}(s) \\ a_{21}(s) & a_{22}(s) \end{bmatrix} \quad (5.45)$$

$$a_{11}(s) = a_{22}(s) = 0.05 \cdot (-25.5s^6 - 792.927s^5 - 8238.3552s^4 - 36006.2932s^3 - 102971.0717s^2 - 185724.7845s - 132739.0367)$$

$$a_{12}(s) = a_{21}(s) = 0.05 \cdot (26.652s^5 + 732.9299s^4 + 5663.5499s^3 + 7912.2992s^2 - 5205.6686s - 0.41644)$$

$$D_7(s) = s^7 + 30.1s^6 + 295.8329s^5 + 1282.7525s^4 + 4963.0643s^3 + 10590.277s^2 + 18051.8717s + 13273.862$$

To derive the coefficients of the numerator and denominator multinomial of third-order reduced system, firstly, the seven poles of the 7th order higher-order system are obtained from the denominator polynomial and written as $\sigma_1 = -1.2396$, $\sigma_{2,3} = -0.9911 \pm 2.2618i$, $\sigma_{4,5} = -0.3802 \pm 3.1887i$, $\sigma_6 = -13.0522$ and $\sigma_7 = -13.067$. Then from the proposed methodology, the MDIs of all four subsystems are obtained from partial fraction, which helps find the cluster centre of the three clusters formed from the seven poles of the system of higher-order. The cluster centres are obtained from the equation (4.9) for real-valued poles and equations (4.11) and (4.12) for complex-valued poles and written as: $\sigma_{c1} = -1.2637$ and $\sigma_{c2,3} = -0.4587 \pm 2.5722i$. The equation of denominator multinomial of the system of 3rd order is hence procured from the three cluster centres as follows:

$$D_3(s) = s^3 + 2.171s^2 + 7.976s + 8.558$$

The genetic algorithm is applied for obtaining the numerator polynomial for the four subsystems by optimizing the ISE. By optimisation, the system of reduced-order is obtained, as shown in equation (5.46).

$$G_3(s) = \frac{1}{D_3(s)} \begin{bmatrix} c_{11}(s) & c_{12}(s) \\ c_{21}(s) & c_{22}(s) \end{bmatrix} \quad (5.46)$$

Such that

$$c_{11}(s) = c_{22}(s) = -1.21s^2 - 3.26s - 4.279 \text{ and } c_{21}(s) = c_{12}(s) = 0.554s^2 - 0.383s + 0.001$$

$D_3(s) = s^3 + 2.171s^2 + 7.976s + 8.558$

So from the IPCG technique, the reduced-order approximation of the system of higher-order of equation (5.45) is procured and shown in equation (5.46). The accuracy of the system of reduced-order of equation (5.46) is checked by comparing it with the higher-order of equation (5.45) based on both systems' time and frequency-based behaviour. The step response behaviour for the complete system is revealed in figure 5.41. The figure shows that the final value of the system of reduced-order of in:(1) out:(1) and in:(2) out:(2) subsystems is similar to the related system of higher-order. The first undershoot of the reduced system is slightly lower than the original system, but the first overshoot of the reduced system is slightly higher. The steady-state of the reduced system is seen to reach approximately the same time.

Table 5.30. Value of errors and various parameters to compare higher 7th order multivariable system and obtained 3rd order system of example 1 for the unit step input

	Para-meter	ISE	ITAE	IAE	OS (%)	RT (sec.)	ST (sec.)	SSE	GM	PM
In 1, Out 1 & in 2, out 2	HOS	-	-	-	52.8	0.297	7.885	-	1.5	19.2
	ROS	0.027	1.483	0.394	39.5	0.342	7.307	0	1.0	2.28
In 1, Out 2 & in 2, out 1	HOS	-	-	-	9.80×10^{6}	3.55×10^{-6}	10.85	-	36	∞
	ROS	8.4×10^{-03}	0.945	0.239	8.19×10^{4}	1.88×10^{-4}	9.132	0	19	∞

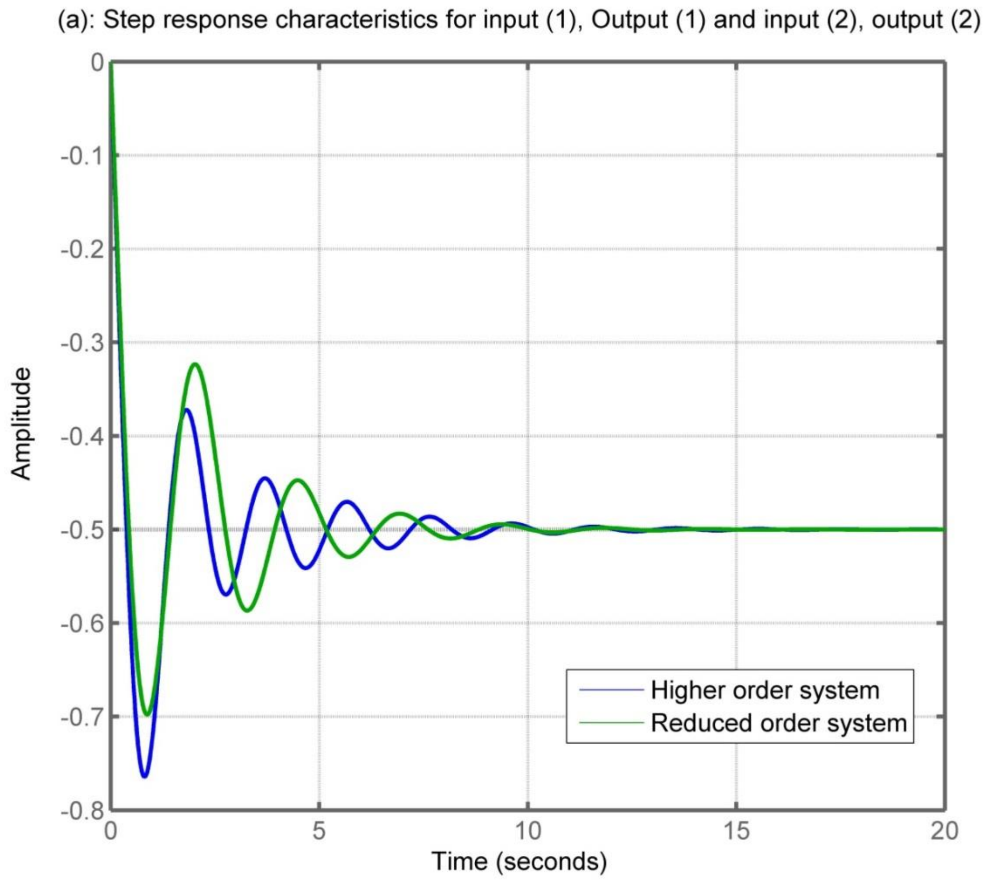


Figure 5.41 (a). Step response characteristics of the MIMO system of higher and lower order obtained by IPCG technique on example 2 for in: (1), out: (1) and in: (2), out: (2)

Similarly, the resultant of the subsystems of reduced-order of in:(1) out:(2) and in:(2) out:(1) is shown in figure 5.41 (b), showing that the response of the system of reduced-order is almost similar to the response of the related system of higher-order following the same path. It is revealed from the figure that the oscillations in the system of reduced-order are reduced with not much change in overshoot and undershoot. It seems that the finalised stage of the system of reduced-order reaches earlier than the system of higher-order. The more accurate conclusion on the step response behaviour can be drawn from Table 5.30. It is clear from Table 5.30 that for in:(1) out:(1) and in:(2) out:(2) subsystem, the value of rising time and time to settle down for the system of higher and reduced-order is similar. The value of steady-state error is observed as zero with the reduction in overshoot in the reduced system compared to a higher-order system. Also, it is noted that the value of ISE is shallow, resulting in an accurate approximation of the system of higher-order. Moreover, the ITAE and IAE are also very low. So, the reduced-order approximation is said to be the replica of a higher-order system. Furthermore, for in:(2) out:(1) and in:(1) out:(2) subsystem, it is seen that there is a minute variation in rising time and time of settling down for the system of reduced-order as procured by IPCG technique.

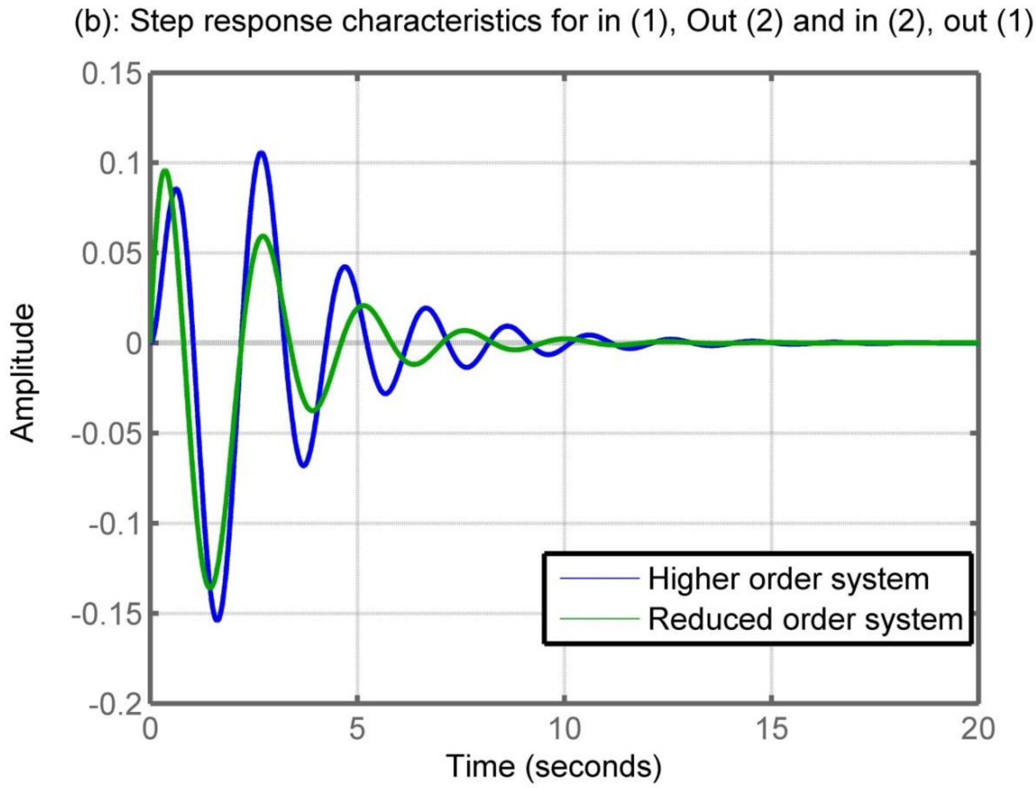


Figure 5.41 (b). Step response characteristics of the multivariable system of higher and reduced-order as procured from IPCG technique on example 2 for in: (1), out: (2) and in: (2), out: (1)

Also, the value of overshoot is slightly lower than that of a higher-order system with zero error at the final time. Moreover, the ISE is also very low, promising high accuracy in the system of lower and higher-order. Also, the low value of ITAE and IAE signifies the high accuracy of the reduced-order approximation. So, the step response comparison in figure 5.41 and Table 5.30 shows that the characteristics of the time-domain of the system of 3rd order approximately match with the higher 7th order system.

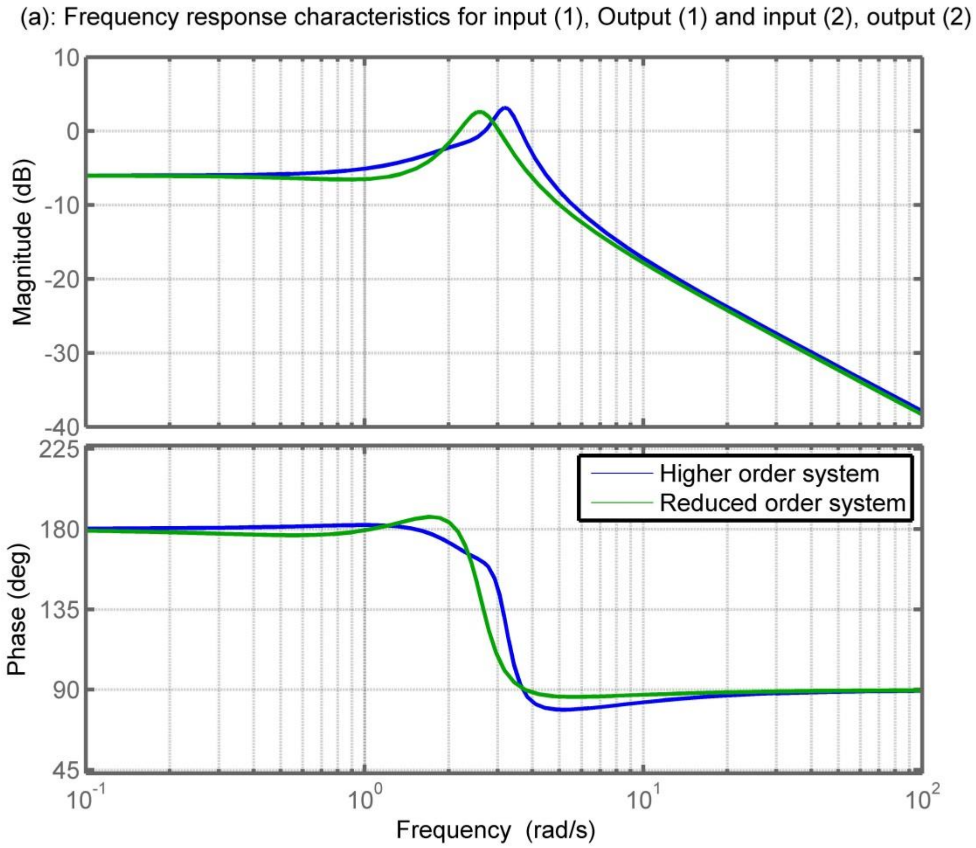


Figure 5.42 (a). Frequency-domain characteristics of the system of higher and reduced-order obtained by IPCG technique on example 2 for in: (1), out: (1) and in: (2), out: (2)

analysing the frequency-based behaviour of the multivariable system of higher and reduced-order from figure 5.42, it is clear that the gain response of the in:(1) out:(1) and in:(2) out:(2) reduced-order subsystems is almost the same with corresponding higher-order systems over the complete frequency range. Furthermore, the phase response of all reduced-order subsystems is also found to be similar to the corresponding higher-order system over the complete frequency range. For the in:(2) out:(1) and in:(1) out:(2) subsystem, the magnitude response of the reduced-order system varies at low frequencies, and there is a slight deviation in phase response also at the low-frequency range. The main characteristic of frequency response, i.e., GM and PM value, is noted from Table 5.30, which results that the value of GM and PM of higher and reduced-order system is approximately the same for all subsystems and have the same sign. So, it has resulted from the frequency-based characteristics that the response in frequency-domain is almost similar for both systems.

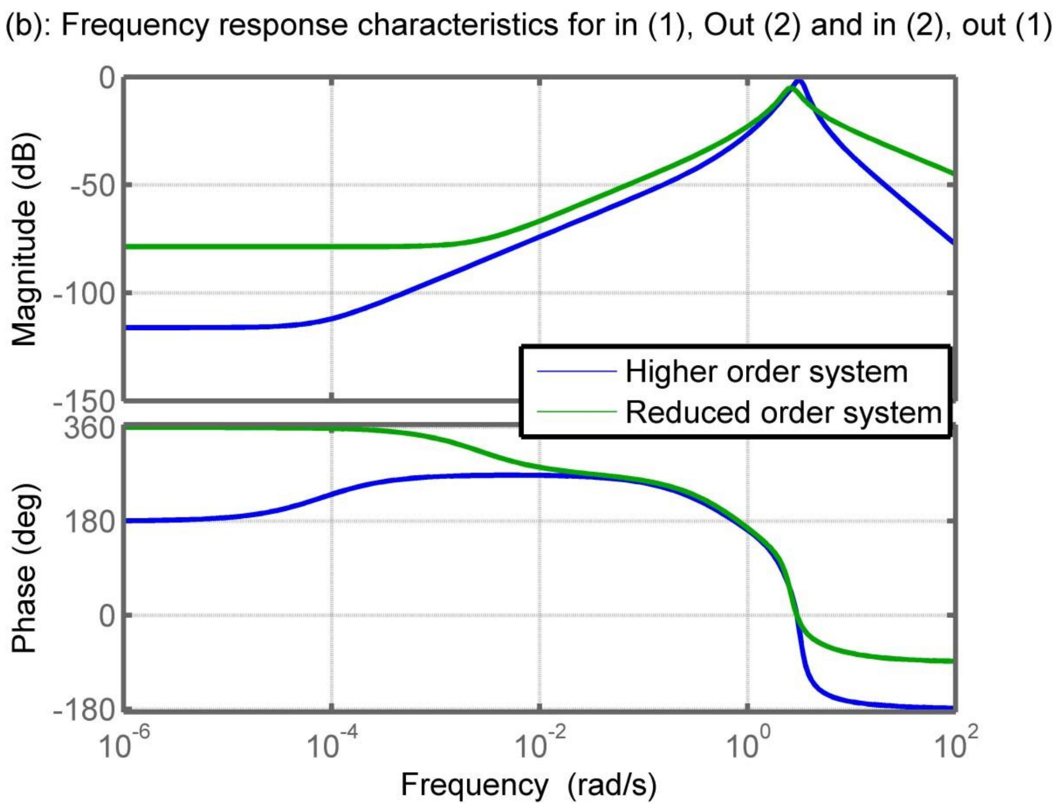


Figure 5.42 (b). Frequency response characteristics of the multivariable system of higher and reduced-order procured from IPCG technique on example 2 for in: (1), out: (1) and in: (2), out: (2)

The equivalency in the system of higher and reduced-order is proved by obtaining the time-moments of both systems. The similar value of the time moments of the two systems results that these systems are similar. The first three-time moments are procured by equation (5.17) by putting the value of *i=1, 2* and *3*. The first three-time moments of the system of higher and reduced-order are obtained in Table 5.31. It shows that the time moments of the system of higher and reduced-order is approximately the same, and hence it results that the given system of 8th order is matching the derived system of 2nd order.

Comparative investigation: For checking the superiority of the implemented technique over other techniques, the comparison of the implemented technique with other techniques (Glover, 1984; Moore, 1981; Vasu et al., 2020) is performed. All techniques are used to reduce the given 7th order multivariable system, and a system of 3rd order is procured. The comparison is conducted based on the step input based characteristics of the system of reduced-order procured by different techniques, as illustrated in figure 5.43. The performance in time-domain and ISE, ITAE and IAE are also utilised for the comparative analysis, as shown in Table 5.32.

Table 5.31. First three time-moments of higher and reduced multivariable system of example 2

Sub-systems	Higher-order system			Reduced-order system		
	First moment	Second moment	Third moment	First moment	Second moment	Third moment
In:(1), Out:(1)	-0.0196	0.0754	0.0941	-0.0851	0.1877	0.7459
In:(2), Out:(1)	-0.0196	0.1129	-0.2389	-0.0449	0.2130	-0.5275
In:(1), Out:(2)	-0.0196	0.1129	-0.2389	-0.0449	0.2130	-0.5275
In:(2), Out:(2)	-0.0196	0.0754	0.0941	-0.0851	0.1877	0.7459

It is analysed from figure 5.43 (a) that the step response of in:(1) out:(1) and in:(2) out:(2) subsystem is obtained by applying all techniques, shows an approximately similar response. The starting value of all responses is at the same amplitude, but the steady-state response shows that the hankel norm and balanced truncation technique show different steady-state values. The value of the first overshoot is almost similar in all the techniques. The more precise comparative investigation for in:(1) out:(1) and in:(2) out:(2) subsystem is performed by obtaining the value of time-domain characteristics and error values and is illustrated in Table 5.32 (a). It results that the value of rising time of all subsystems of reduced-order is approximately matching to the related subsystems of higher-order. Similarly, the time of settling down in the IPCG technique is most similar to that of the original system. Moreover, the overshoot possessed by all techniques varies from the original system.

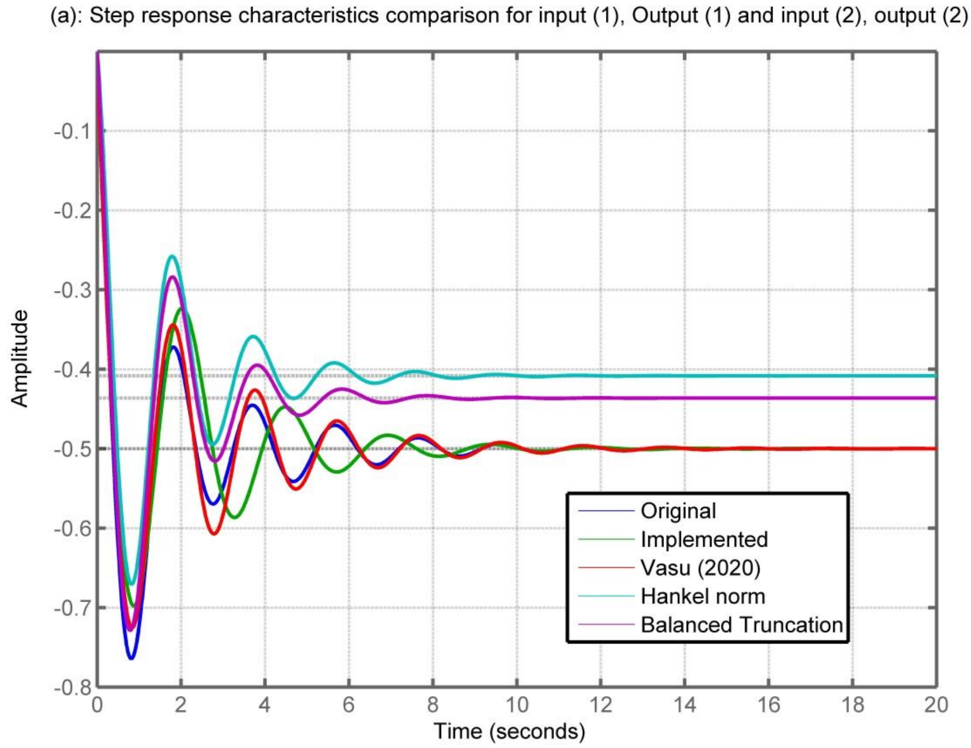


Figure 5.43 (a). Comparative investigation of IPCG technique with literature techniques for the reduction of the system of 7th order based on step response characteristics for 1st input, 1st output and 2nd input, 2nd output combination

Table 5.32 (a). Comparative analysis of errors and time-domain performance parameters for 1st input, 1st output and 2nd input, 2nd output subsystems of 7th order multivariable system of example 2 subjected to the unit step input

Parameter	ISE $\times 10^{-05}$	ITAE	IAE	OS (%)	RT (sec.)	ST (sec.)	SSE
Original	-	-	-	52.8	0.297	7.885	-
Implemented	0.027	1.483	0.394	39.5	0.342	7.307	0
Vasu (2020)	0.092	4.75	0.948	45.5	0.316	8.799	0
Hankel norm approximation	0.1705	18.350	1.838	64.12	0.303	6.764	0.1
Balanced Truncation	0.0811	12.741	1.249	67.08	0.272	6.054	0.06

Also, the error in the final stage is zero for implemented technique and technique of Vasu. From the analysis of errors, it is clear that the value of ISE is the lowest for implemented technique. At the same time, the value of ITAE and IAE is also the lowest for implemented technique. So, the comparative study results that the reduction technique presented by implemented technique is better than other techniques developed in the literature. It is analysed from figure 5.43 (b) that the step response of in:(1) out:(2) and in:(2) out:(1) subsystem is obtained by applying all techniques, shows approximately similar step response.

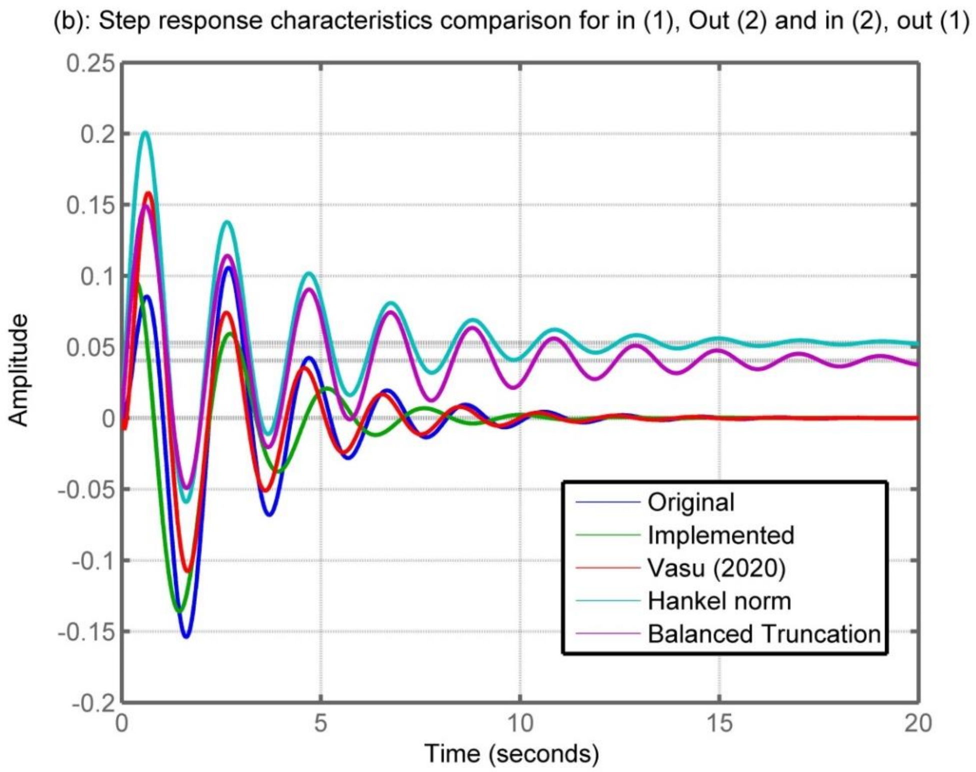


Figure 5.43 (b). Comparative analysis of newly implemented technique with techniques given in the literature for the reduction of 7th order multivariable system based on step response characteristics for 1st input, 2nd output and 2nd input, 1st output combination

The starting of all responses is at the same amplitude, but the steady-state response shows that the Hankel norm and balanced truncation technique show different steady-state values. The value of the first overshoot is almost similar in all the techniques other than the Hankel norm approximation. The more precise comparative analysis for in:(1) out:(2) and in:(2) out:(1) subsystem is performed by obtaining the value of time-domain characteristics and error values and is illustrated in Table 5.32 (b). It clears that the value of rising time of all subsystems of reduced-order is approximately the similar to the related subsystems of higher-order. And the settling time of the system of reduced-order procured by the IPCG technique matches more exactly to the original system than literary techniques. Moreover, the overshoot possessed by all techniques varies from the original system. Also, the error at the final time is zero for only implemented technique. From the analysis of errors, it is clear that the value of ISE is lowest for implemented technique, and the value of ITAE and IAE is also the lowest for implemented technique. So, the comparative study results that the reduction technique presented by implemented technique is better than other techniques developed in literature and used for comparative analysis.

Table 5.32 (b). Comparative analysis of errors and time-domain performance parameters for 2nd input, 1st output and 1st input, 2nd output subsystems of 7th order multivariable system of example 2 subjected to the unit step input

Parameter	ISE $\times 10^{-05}$	ITAE	IAE	OS (%)	RT (sec.)	ST (sec.)	SSE
Original	-	-	-	9.80 $\times 10^{6}$	3.55 $\times 10^{-6}$	10.855	-
Implemented	8.4 $\times 10^{-03}$	0.945	0.239	8.19 $\times 10^{4}$	1.88 $\times 10^{-4}$	9.132	0
Vasu (2020)	0.070	4.870	1.761	6.86 $\times 10^{6}$	4.86 $\times 10^{-6}$	10.61	0.0052
Hankel norm approximation	0.685	10.670	11.32	278.19	0.081	15.049	∞
Balanced Truncation	0.421	8.135	8.631	269.54	0.087	21.170	∞

From the reduction of the 7th order MIMO system to procure the system of 3rd order by the IPCG method in the research work, it is clear that the implemented method provides a most matched approximation of the system of higher-order with approximately similar time and frequency domain characteristics and equivalent time moments. Also, the implemented technique provides a stable reduced-order system. Furthermore, the technique implemented here also provides better time-domain characteristics for step input than other techniques presented in the literature.

$$A=\begin{bmatrix}
0 & 0 & 0 & 376.9911 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 376.9911 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 376.9911 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-0.1029 & 0.0750 & -0.0277 & -0.2689 & -0.0189 & -0.0189 & -0.0018 & -0.0019 & -0.0019 & -0.0019 & -0.0019 & -0.0019 & -0.0019 & -0.0019 & -0.0018 & -0.0019 & -0.0019 & -0.0019 & -0.0122 \\
0.1200 & -0.1242 & -0.0411 & -0.0173 & -0.2737 & -0.0173 & -0.0017 & -0.0017 & -0.0017 & -0.0017 & -0.0017 & -0.0017 & -0.0017 & -0.0017 & -0.0017 & -0.0017 & -0.0017 & -0.0017 & -0.0126 \\
0.0114 & 0.0137 & -0.1814 & -0.0019 & -0.0019 & -0.2876 & -0.0002 & -0.0002 & -0.0002 & -0.0002 & -0.0002 & -0.0002 & -0.0002 & -0.0002 & -0.0002 & -0.0002 & -0.0002 & -0.0002 & -0.0050 \\
-0.2688 & 15.0787 & -6.5030 & 0.4818 & 0.4818 & 0.4818 & -99.953 & 0.0482 & 0.0482 & 0.0482 & 0.0477 & 0.0482 & 0.0482 & 0.0482 & 0.0468 & 0.0482 & 0.0482 & 0.0482 & -0.2796 \\
0 & 0 & 0 & 0.0500 & 0 & 0 & 0 & -0.0500 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -187.50 & 0 & 0 & 0 & 187.50 & -25.000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -562.50 & 0 & 0 & 0 & 562.50 & -25.000 & -25.000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-14.248 & -5.5473 & -36.800 & -23.557 & -23.557 & -23.557 & -2.2871 & -2.3557 & -2.3557 & -2.3557 & -102.33 & -2.3557 & -2.3557 & -2.3557 & -2.2871 & -2.3557 & -2.3557 & -2.3557 & -15.205 \\
0 & 0 & 0 & 0 & 0.0500 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0500 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -187.50 & 0 & 0 & 0 & 0 & 0 & 0 & 187.50 & -25.000 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -562.50 & 0 & 0 & 0 & 0 & 0 & 0 & 562.50 & -25.000 & -25.000 & 0 & 0 & 0 & 0 & 0 \\
3.0311 & 18.4615 & -52.958 & -16.639 & -16.639 & -16.639 & -1.6155 & -1.6640 & -1.6640 & -1.6640 & -1.6475 & -1.6640 & -1.6640 & -1.6640 & -101.61 & -1.6640 & -1.6640 & -1.6640 & -8.9087 \\
0 & 0 & 0 & 0 & 0 & 0.0500 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0500 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -187.50 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 187.50 & -25.000 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -562.50 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 562.50 & -25.000 & -25.000 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -43.600
\end{bmatrix}$$

$$B=\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 43.600 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 43.600
\end{bmatrix}^T$$

$$C=\begin{bmatrix}
0.8950 & 0.2380 & 0.0270 & 0.0510 & 0.0510 & 0.0510 & 0.0050 & 0.0050 & 0.0050 & 0.0050 & 0.0050 & 0.0050 & 0.0050 & 0.0050 & 0.0050 & 0.0050 & 0.0050 & 0.0050 & 0.0440 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

$$D=\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad (5.47)$$

Example 3: Consider a 19th order multivariable system consisting of two inputs and four outputs as represented in state-space representation as depicted in equation (5.47).

To derive the numerator and denominator coefficients of the system of fifth-order, firstly, it is required to procure the poles of the 19th order system. Then according to the value of MDIs, the reduced-order denominator polynomial is procured as follows:

$$D_5(s) = s^5 + 0.4108s^4 + 9.836s^3 + 1.471s^2 + 0.07345s + 0.001224$$

The genetic algorithm obtains the numerator polynomial for the eight subsystems by optimizing the value of ISE. Through optimisation, the system of reduced-order is procured for both inputs separately. For the first input, the reduced-order system is represented in state-space form as:

$$A_{r1} = \begin{bmatrix} -0.4108 & -2.459 & -0.3678 & -0.1469 & -0.03917 \\ 4 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0.125 & 0 & 0 \\ 0 & 0 & 0 & 0.0625 & 0 \end{bmatrix}$$

$$B_{r1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

$$C_{r1} = \begin{bmatrix} 0.5495 & 1.043 & 0.1547 & 0.0616 & 0.01641 \\ -0.0264 & -0.000525 & -2.603\times10^{-05} & -3.48\times10^{-06} & -2.56\times10^{-19} \\ -0.01548 & -0.0005822 & -2.912\times10^{-05} & -3.884\times10^{-06} & -2.202\times10^{-19} \\ -0.00131 & -4.95\times10^{-05} & -2.48\times10^{-06} & -3.311\times10^{-07} & -2.193\times10^{-20} \end{bmatrix}$$

$$D_{r1} = 0 \qquad (5.48)$$

Similarly, for the second input, the state space representation of the 5th order reduced system is:

$$A_{r2} = \begin{bmatrix} -0.4108 & -2.459 & -0.3678 & -0.1469 & -0.03917 \\ 4 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0.125 & 0 & 0 \\ 0 & 0 & 0 & 0.0625 & 0 \end{bmatrix}$$

$$B_{r2} = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

$$C_{r2} = \begin{bmatrix} 0.07 & -0.7079 & -0.1065 & -0.0426 & -0.01137 \\ 0.198 & -1.229 & 0.1839 & 0.7345 & 0.01958 \\ -0.00774 & -0.0002911 & -1.456\times10^{-05} & -1.942\times10^{-06} & -1.101\times10^{-19} \\ -0.000655 & -2.478\times10^{-05} & -1.24\times10^{-06} & -1.655\times10^{-07} & -1.097\times10^{-20} \end{bmatrix}$$

$$D_{r2} = 0 \qquad (5.49)$$

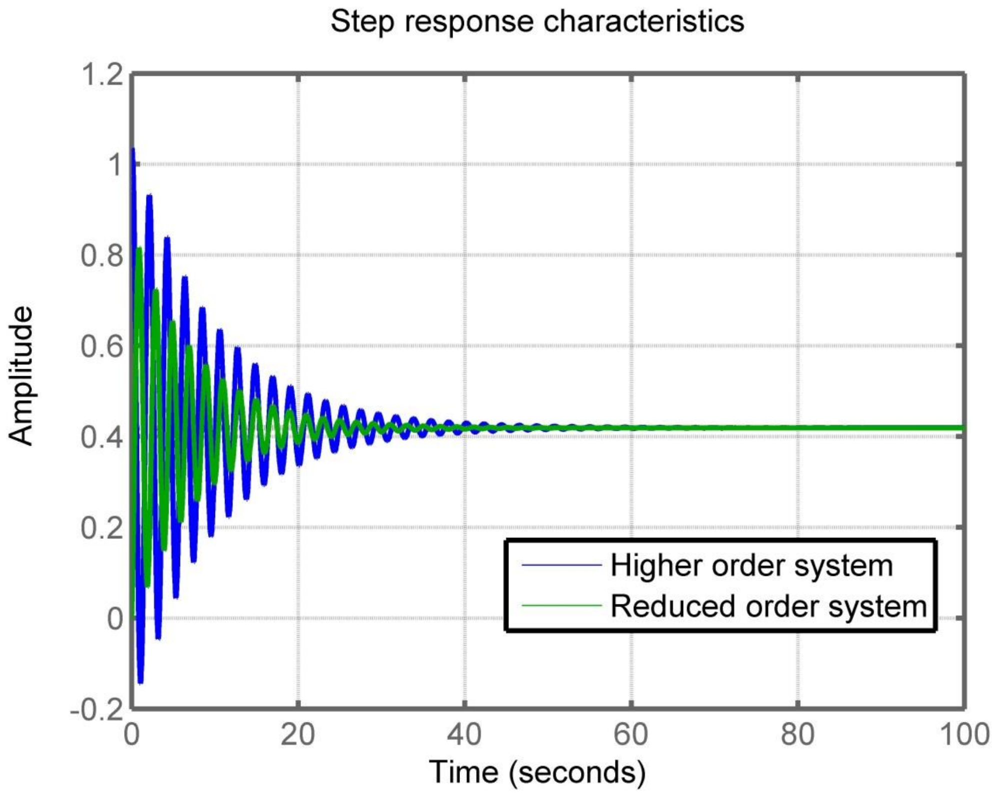


Figure 5.44. Response of the multivariable system of high and low order obtained from implemented technique on example 3 for in: (1), out: (1) for the unit step input

So, the approximation of reduced-order procured from the 19th order system of equation (5.47) is obtained by implemented technique and presented in equation (5.48) and equation (5.49) for first and second input, respectively. The accuracy of the system of reduced-order is checked by comparing both systems based on their step response and frequency response behaviour. The step response behaviour is shown in figure 5.44, which depicts that the response of the reduced-order subsystem for in:(1) out:(1) is approximately similar to that of the corresponding system of higher-order. The overshoot of the system of reduced-order is a little lower than the original system, and steady-state values are also appearing as similar. The more accurate step response comparison is analysed by obtaining the values of step response performance parameters as given in Table 5.33.

Table 5.33. Value of errors and various parameters to compare higher 7th order multivariable system and obtained 3rd order system of example 2 for the unit step input

Parameter	ISE	ITAE	IAE	OS (%)	RT (sec.)	ST (sec.)	SSE	G M	P M
HOS	-	-	-	147	0.309	38.21	-	∞	332.3
ROS	1.861	37.59	4.62	94.4	0.299	30.14	0	∞	39.3

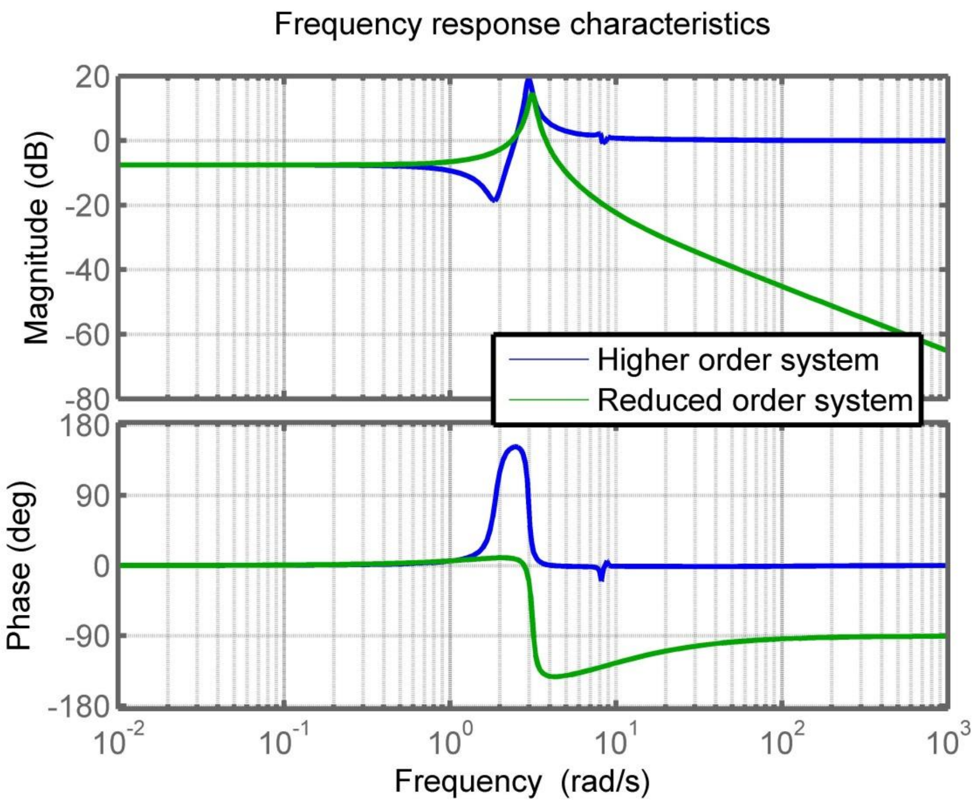


Figure 5.45. Frequency-based characteristics of the multivariable system of high and low order obtained from implemented technique on example 3 for in: (1), out: (1)

It is clear from Table 5.33 that for in:(1) out:(1) subsystem, the value of time to rise and time to settlement for both systems is almost similar. The value of error at the final time is observed as zero with a slight reduction in the overshoot in the reduced system compared to a higher-order system. Also, it is noted that the value of ISE is shallow, resulting in an accurate approximation of the system of higher-order. Moreover, ITAE and IAE are also observed as very low. So, the system of reduced-order procured by implemented technique is a sound reduction of the system of higher-order based on the step response performance. analysing the frequency response behaviour of the multivariable system of high and low order from figure 5.45, the magnitude response of the in:(1) out:(1) reduced-order subsystem is almost the same as the corresponding higher-order system over the complete frequency range. Furthermore, the phase response of all reduced-order subsystems is also found to be almost similar to the corresponding higher-order system at lower frequencies. In comparison, at higher frequencies, there is a variation in the phase of the system of reduced-order. The main characteristic of frequency response, i.e., GM and PM, is noted from Table 5.33, resulting in the value of GM and PM of higher and reduced-order systems having the same polarity. It signifies that the stability behaviour of the higher system remains the same after reduction. So, the frequency characteristics result that the multivariable system of a high order is almost similar to the system of reduced-order obtained by implemented IPCG technique.

The equivalency in the system of high and low order is also proved by fetching the time moments of both systems. The similar value of the time moments of the two systems results that these systems are similar. The first three time-moments are obtained from equation (5.17) by putting the value of $i=1, 2$ and 3. Hence, the first three time-moments of the system of high order for the first input first output subsystem are obtained as $T_{l1}= 0.0406$, $T_{l2}= -0.7873$ and $T_{l3}= 99.1146$ and the first three time-moments of the system of the reduced order are $T_{r1}= 0.0178$, $T_{r2}= 0.0434$ and $T_{r3}= 128.529$. It shows that the time moments of the system of the higher-order and low order as procured by the IPCG method is approximately the same. Hence it results that the given 8^{th} order system shows similarity to the derived 2^{nd} order system.

Comparative investigation: For justifying the superiority of the implemented IPCG technique over other techniques, the comparison of the implemented technique with other techniques (O. Alsmadi et al., 2019; Glover, 1984; C. N. Singh et al., 2019) is performed. All techniques are used to reduce the given in:(1) out:(1) subsystem of 19th order multivariable system, and a 5th order system is procured. The comparison is conducted based on the step response of the system of reduced-order procured by different techniques, as drawn in figure 5.46. The performance characteristics in the time domain and ISE, ITAE and IAE are also utilised for the comparative analysis as shown in Table 5.34, which clears that all techniques show the same behaviour. The amplitude of oscillations is different, and they are displaced by each other from an offset value. The most accurate reduced-order approximation is obtained by implemented technique and the technique given by Singh. The technique of Alsmadi and Glover provides a fixed error in the whole time range, and hence, the steady-state error is increased in these two techniques.

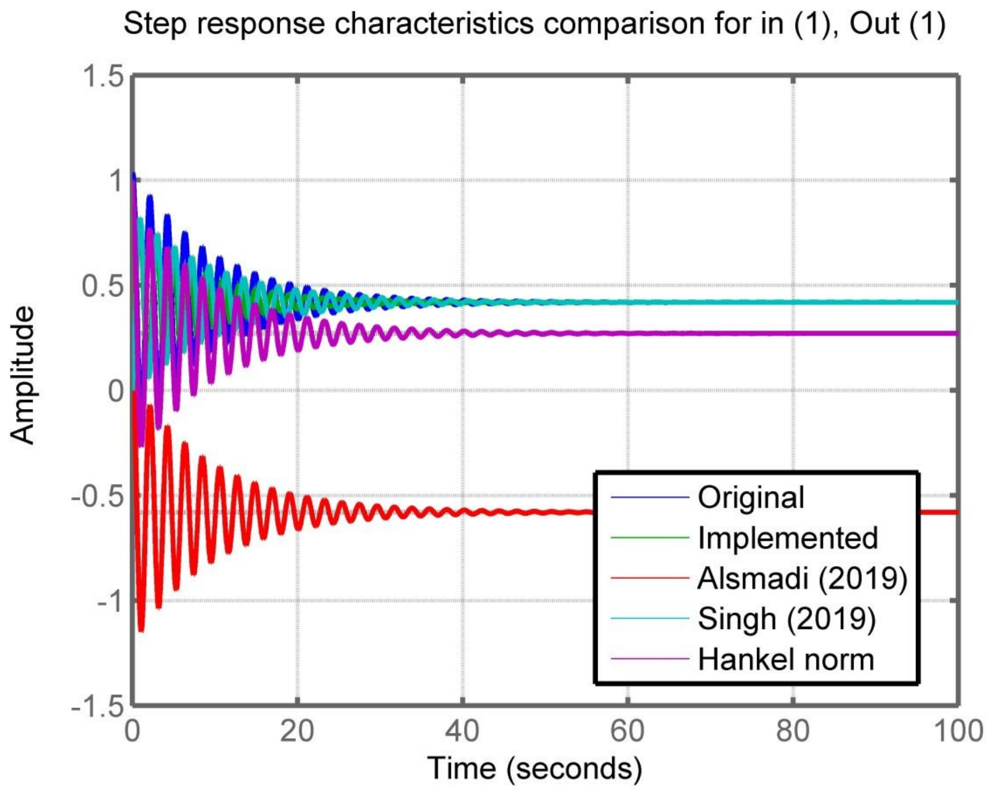


Figure 5.46. Comparative analysis of newly implemented technique with techniques given in literature reducing the 19th order multivariable system based on step response characteristics for 1st input, 1st output subsystem

For getting the exact value of time-domain performance parameters, Table 5.34 is analysed. From the table, it is justified that the value of rising time is better for the implemented technique. Still, the value of settling time shows that the response of implemented technique reaches the steady-state at a faster rate than other techniques. The value of error at steady state is observed as zero in implemented technique and technique of Singh only. Moreover, the value of overshoot provided by all responses is different from the original system. So, implemented technique and Singh's technique are suggested better than all other techniques based on time-domain parameters. From the analysis of error value in Table 5.34, it is clear that the value of ISE is least in implemented technique as compared to all other techniques. Moreover, while analysing the value of ITAE and IAE, the implemented technique provides a closer response of reduced-system compared to other techniques. So, it reveals that the implemented technique provides a better approximated system of reduced order from the 19th order system than other techniques.

Table 5.34. Comparative analysis of errors and time-domain performance parameters for 1st input, 1st output subsystem of 19th order multivariable system of example 3 subjected to the unit step input

Parameter	ISE $\times 10^{-05}$	ITAE	IAE	OS (%)	RT (sec.)	ST (sec.)	SSE
Original	-	-	-	147	0.309	38.21	-
Implemented	1.861	37.59	4.62	94.4	0.299	30.14	0
Alsmadi (2019)	99.91	499.7	99.96	97.39	0.358	38.17	0.17
Singh (2018)	2.623	57.25	6.275	95.67	0.320	39.74	0
Hankel norm approximation	2.184	739.89	14.77	268.8	0.431	37.10	0.14

From the reduction of 19th order multivariable system by the IPCG method implemented in the research work, it is clear that the implemented method provides a sound approximated system of reduced-order from the system of higher-order system with approximately similar time and frequency domain characteristics and equivalent time moments. Also, the implemented technique provides the same stability performance for a reduced-order system as provided by the original system. Furthermore, the technique implemented here also provides better time-domain characteristics for step input than other techniques presented in the literature.

5.4 DESIGN OF CONTROLLER

The first three sections show that the implemented methodology provides a good approximation of higher-order systems. So, this technique can be used for the designing of the controller for power systems. In this section, a controller is designed to remove the effect of load disturbance and, hence, design an LFC controller for power systems having up to three control areas.

Example 1: A power system with a single area of the reheated type hydrothermal system is presented in figure 5.47.

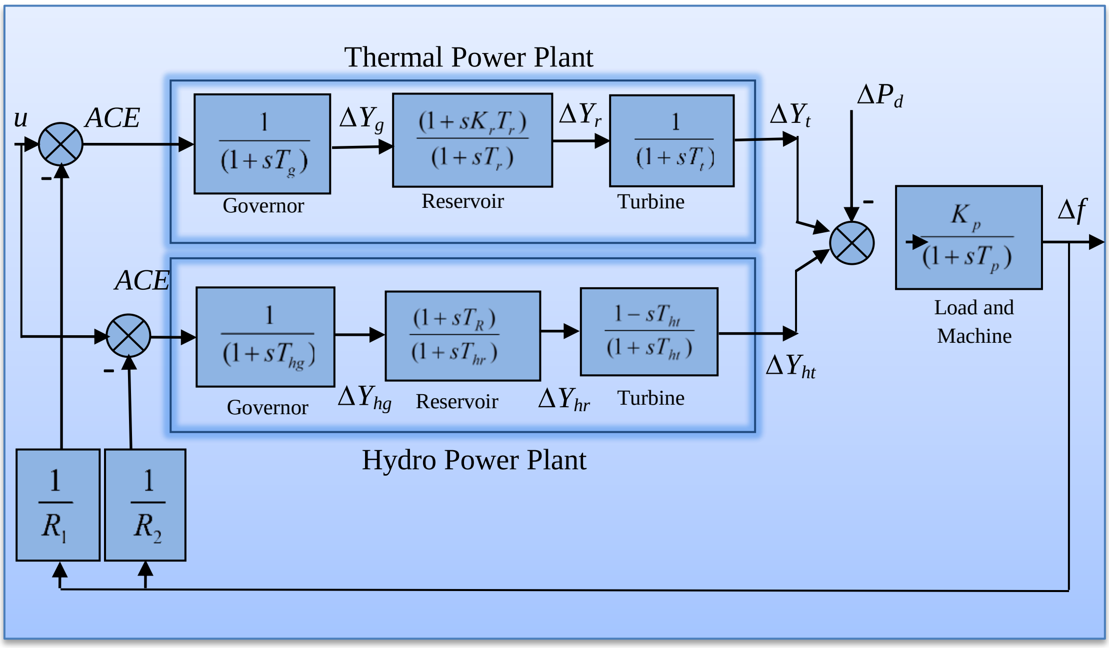


Figure 5.47. Single area reheated thermal-hydropower system in block diagram representation

Table 5.35. Nomenclature and parameter values of single area thermal-hydropower system

Symbol	Nomenclature	Value/ Unit
ΔY_g	The output power of the governor in thermal plant in power system	MW
ΔY_r	The output power of reservoir in the thermal plant of power system	MW
ΔY_t	The output power of turbine in the thermal plant of power system	MW
ΔY_{hg}	The output power of the governor in hydro plant in power system	MW
ΔY_{hr}	The output power of reservoir in the hydro plant of power system	MW
ΔY_{ht}	The output power of turbine in the hydro plant of power system	MW
ΔP_d	Load disturbance	-
ACE	Area Control Error	-
u	Applied input	-
T_g	The time constant of the thermal generator	0.08 seconds
T_t	The time constant of the thermal turbine	0.3 seconds
T_r	The time constant of the thermal reservoir	10 seconds
K_r	Power fraction of high-pressure turbine	0.5
K_p	Gain of Power system	120 Hz/puMW
T_p	Power system time constant	20 seconds
R_1, R_2	Speed regulation of governors	2.4 Hz/puMW
T_{hg}	hydro generator time constant	41.6 seconds
T_{ht}	Water turbine constant	1 second
T_{hr}	Hydro reservoir time constant	0.513 seconds
T_R	Hydro governor time constant	5 seconds

It shows that the tested system is a parallel combination of a hydropower generation system and a thermal power generation system. Both of the power generation systems have a governor, a reservoir for providing reheating and a turbine. So, these two systems are of reheated type. The power system's parameters are described in Table 5.35 with the terminology of all parameters. The figure depicts that the system's output has the effect of two inputs: one is applied input *'u'*, and another is load disturbance 'ΔP_d' in the system. The output of the system is the frequency deviation Δf. Due to the load disturbance in the system, there is a change in frequency of operation, as illustrated by figure 5.48. This frequency change persists in the system. So the aim of implementing a controller is to reduce the change in frequency and remove the effect of load variation on the power system. Hence, a 2-DOF-IMC-PID controller is added in the feedback of the original power system.

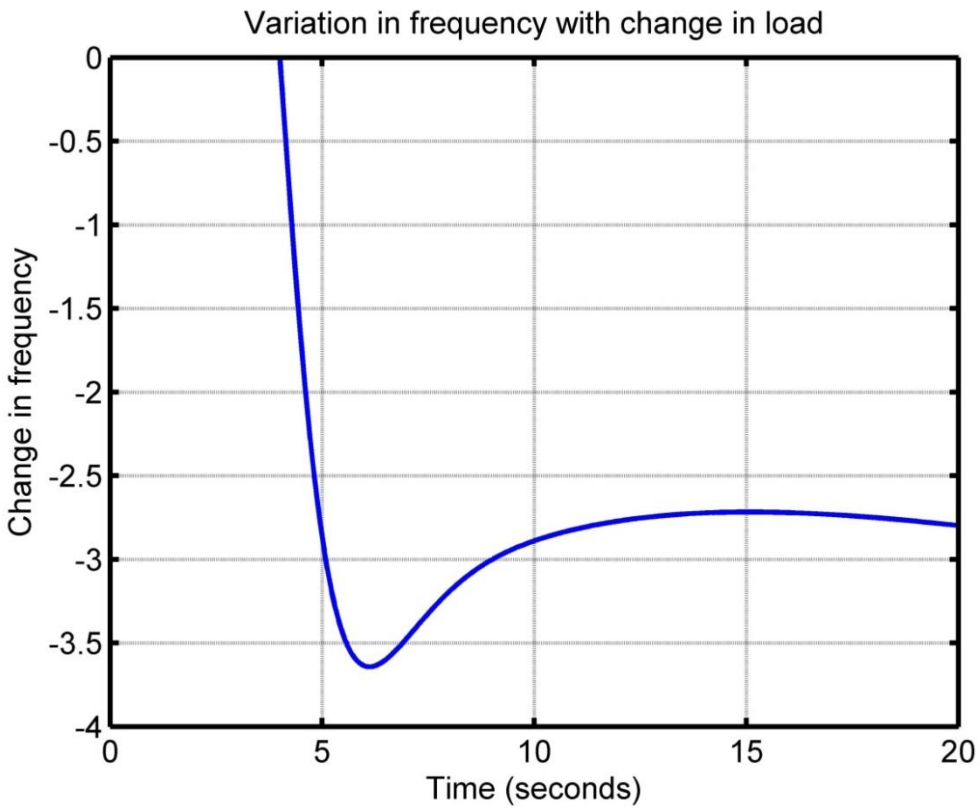


Figure 5.48. Change in frequency with variation in load at 4 seconds for uncompensated reheated hydrothermal power system consisting of the single control area

After placing the parameter values from Table 5.35, the transfer function of the reheat hydrothermal power system consisting of a single control area is procured as depicted in equation (5.52). This equation is formed by combining the equation (5.50) depicting the transfer function for the applied input and equation (5.51) depicting the transfer function related to disturbance input, derived as follows:

$$G_i(s) = \frac{Y_l(s)}{R(s)} = \frac{-144s^5 + 4223s^4 + 22356s^3 + 35496s^2 + 7439s + 240}{51.22s^7 + 1022s^6 + 5653s^5 + 14302s^4 + 19697s^3 + 16338s^2 + 3173s + 101} \quad (5.50)$$

$$G_d(s) = \frac{Y_d(s)}{P_d(s)} = \frac{-307.3s^6 - 6118s^5 - 33972s^4 - 73680s^3 - 58620s^2 - 6359s - 120}{51.22s^7 + 1022s^6 + 5653s^5 + 14302s^4 + 19697s^3 + 16338s^2 + 3173s + 101} \quad (5.51)$$

The equations (5.50) and (5.51) are combined to form the overall transfer function having the effect of both inputs of the system $G_l(s)=G_i(s)+G_d(s)$ and written as follows:

$$G_l(s) = \frac{-6s^6 - 122.2s^5 - 580.8s^4 - 1002s^3 - 451.6s^2 + 21.09s + 2.343}{s^7 + 19.95s^6 + 110.4s^5 + 279.6s^4 + 384.6s^3 + 319s^2 + 61.93s + 1.972} \quad (5.52)$$

The system is of 7th order having poles $\sigma_1 = -12.9076$, $\sigma_{2,3} = -2.7089 \pm 0.8579i$, σ_4, $\sigma_5 = -0.6871 \pm 1.3412i$, $\sigma_6 = -0.2109$, *and* $\sigma_7 = -0.0395$. By using the implemented technique of model order reduction, the 7th order system of equation (5.52) is to be reduced to a 3rd system. So, there will be three clusters in the approximated reduced-order system. One cluster has all real poles, and the cluster centre of this cluster is obtained from equation (4.9) as $\sigma_{c1} = -0.0640$ and the second and third cluster contains the complex poles and hence resulted in the cluster centre σ_{c2}, $\sigma_{c3} = -0.8093 \pm 0.9982i$. Now, the genetic algorithm is applied to obtain the numerator coefficients. After 69 iterations of the genetic algorithm, the reduced-order system of 3rd order is obtained, as shown in equation (5.53).

$$G_3(s) = \frac{-7.81s^2 - 0.709s + 0.124}{s^3 + 1.683s^2 + 1.755s + 0.1027} \quad (5.53)$$

The comparison of the system of high order and the system of reduced-order with the application of step signal at the input is exhibited in figure 5.49. The comparison of responses shows that a better-approximated system of reduced-order is procured from the system of high order with good matching in the responses. The characteristics comparison in the time domain is depicted in Table 5.36. It shows that the errors amongst both systems are procured as ISE is 0.2613, ITAE is 5.15, and IAE is 1.29.

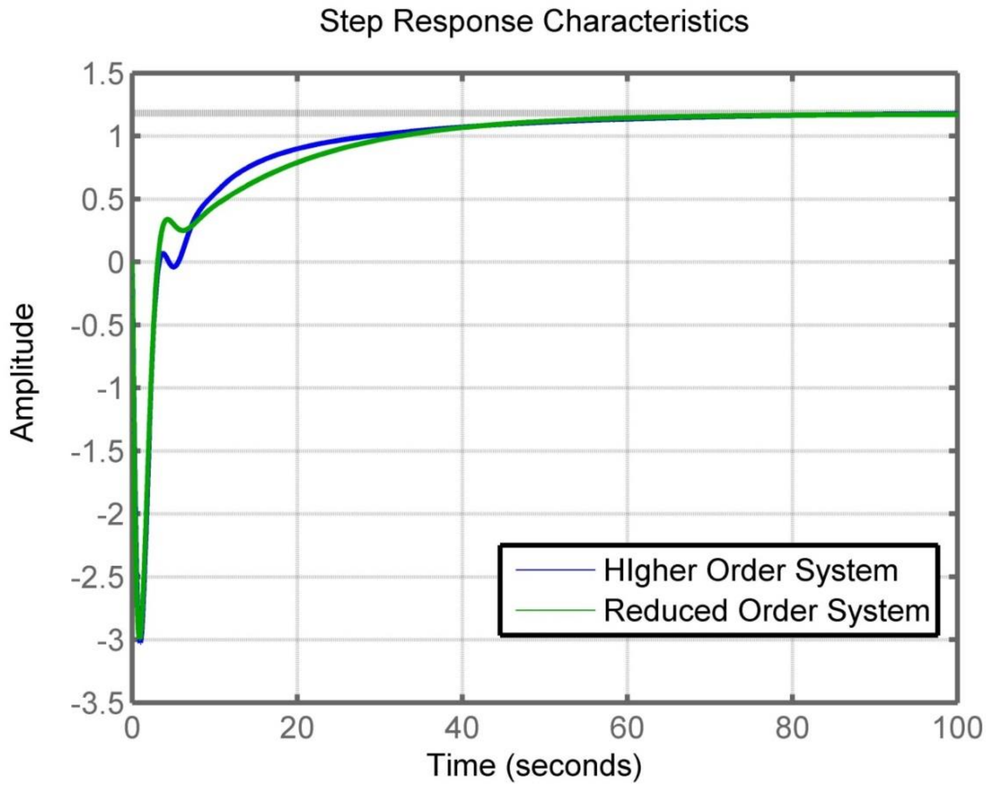


Figure 5.49. Step response characteristics of 7th order single area power thermal-hydropower system and the 3rd order reduced system procured using implemented technique

Table 5.36. Value of errors and various parameters to compare the higher-order system of single area thermal hydropower system and obtained 3rd order system of example 1 for the unit step input

Parameter	ISE $\times 10^{-05}$	ITAE	IAE	OS (%)	RT (sec.)	ST (sec.)	SSE
HOS	-	-	-	0	33.4671	48.4078	-
ROS	0.2613	6.1575	1.2944	0	36.4590	45.6197	0

The value of errors shows that the system of reduced-order is similar to the system of a high order. Also, the value of transient response characteristics shows that the difference in rising time, time of reaching final value and maximum value between the system of high and reduced-order is significantly less, and the error at steady state is reduced to zero. So, it has resulted that the reduced-order system derived from implemented technique exhibits the same behaviour of time-domain characteristics as shown by the higher-order system. Hence, the approximated reduced-order system of equation (5.53) is used to design a controller instead of the system of high order given in equation (5.52).

As described in the methodology, the steps to design the LFC controller are performed as follows:

1. *Factorization:* The reduced-order model $G_3(s)$ is factorized into two parts: minimal phase, i.e. invertible portion and non-minimal phase, i.e. non-invertible portion, which is represented in equation (5.54) as follows:

$$G_3(s) = G_3^-(s)\cdot G_3^+(s) = \left[\frac{-7.81s^2 - 0.709s + 0.124}{s^3 + 1.683s^2 + 1.755s + 0.1057}\right]\cdot 1 \tag{5.54}$$

2. *Inversion:* The invertible portion is then inverted, and the non-invertible part is kept intact to get the compensator. Hence, the equation for compensator is obtained as follows:

$$Q_{c1}(s) = \left[\frac{-7.81s^2 - 0.709s + 0.124}{s^3 + 1.683s^2 + 1.755s + 0.1057}\right]^{-1}\cdot 1 \tag{5.55}$$

3. *Filtration:* For fetching the strictly proper IMC compensator for the single area thermal-hydropower system of equation (5.52), a low pass filter is added in the cascade of the compensator obtained in equation (5.55). The filtered IMC compensator of equation (5.56) is derived for the first-order low pass filter as follows:

$$\begin{aligned} Q_{c1}(s) &= \left[G_3^-(s)\right]^{-1}\cdot G_3^+(s)\cdot F(s) \\ &= \left(\frac{-7.81s^2 - 0.709s + 0.124}{s^3 + 1.683s^2 + 1.755s + 0.1057}\right)^{-1}\cdot\left(\frac{1}{\lambda s + 1}\right) \end{aligned} \tag{5.56}$$

The value of *'λ'* is to be procured by minimising the error function ISE for the optimised performance of the compensator of the IMC type. After optimisation, the value of *'λ'* is obtained as $\lambda=0.291$ after eight generations. Hence, placing the value of

'λ' in equation (5.56), the IMC compensator can be designed as shown by equation (5.57) as follows:

$$\begin{aligned} Q_{c1}(s) &= \left(\frac{-7.81s^2 - 0.709s + 0.124}{s^3 + 1.683s^2 + 1.755s + 0.1057} \right)^{-1} \cdot \left(\frac{1}{0.291s + 1} \right) \\ &= \frac{s^3 + 1.683s^2 + 1.755s + 0.1057}{-2.2727s^3 - 7.6037s^2 - 0.7451s + 0.124} \end{aligned} \tag{5.57}$$

4. For procuring the 2-DOF IMC compensator, the second compensator is obtained by the procedure described in the equation (4.25). The value of *'m'* is chosen equal to 1 because there is only one pole experiencing the effect of load disturbance and is to be cancelled. Hence after the cancellation of pole $s=-T_p$, the value of λ_2 is obtained as *0.5* and k_1 is obtained as *0.216*. Thus, the disturbance rejecting IMC compensator equation (5.58) is presented as follows:

$$Q_{c2}(s) = \frac{1 + 0.2s}{1 + 0.5s} \tag{5.58}$$

The two compensators presented in equation (5.57) and equation (5.58) designed by the implemented methodology are then combined to form the 2-DOF IMC compensator. The 2-DOF-IMC compensator is then added in the feedback of the hydrothermal power system consisting single control area, and the controller of IMC type is obtained as follows:

$$C(s) = \frac{Q_{c1}(s).Q_{c2}(s)}{1 - G_3(s) \cdot Q_{c1}(s).Q_{c2}(s)} = \frac{s^4 + 6.314s^3 + 9.416s^2 + 8.231s + 0.489}{-5.259s^4 - 0.731s^3 - 5.953s^2 - 0.409s} \tag{5.59}$$

Equation (5.59) is a feedback controller obtained from the approximated system of reduced-order procured by reducing the order of the system of the high order of equation (5.52). This controller is then utilised to procure the PID controller's gain parameters (K_p, K_i, K_d) from the least square model matching method given in equation (5.30). The solution of equation (5.30) results in the following value of gain parameters:

$$K_p=2.937,\ K_i=3.999,\ K_d=1.0956 \tag{5.60}$$

The effectiveness of the IMC-PID controller designed in equation (5.60) on the single area reheated hydrothermal system can be studied from figure 5.50. Figure 5.50 shows the variation in frequency due to deviation in load. Different intensities of load variation cause different values of undershoot occurring in frequency. More change in load causes more value of undershoot in frequency. It is depicted from the figure that when variation in load is 1%, frequency experiences the undershoot value equal to 0.016. The undershoot increases with increasing load variation and exhibits a value of 0.16 for 10% load variation. Also, it is to note here that the settling time and the number of oscillations for all the cases of load variation are the same, i.e. 4 seconds, whether the load variation is 1% or 10%. So from figure 5.50, it has resulted that the increase in load variation increases the deviation in frequency, but the settling time and number of oscillations do not differ. Hence, increasing the load variation increases undershoot of the compensated hydrothermal system and keeps settling time unchanged.

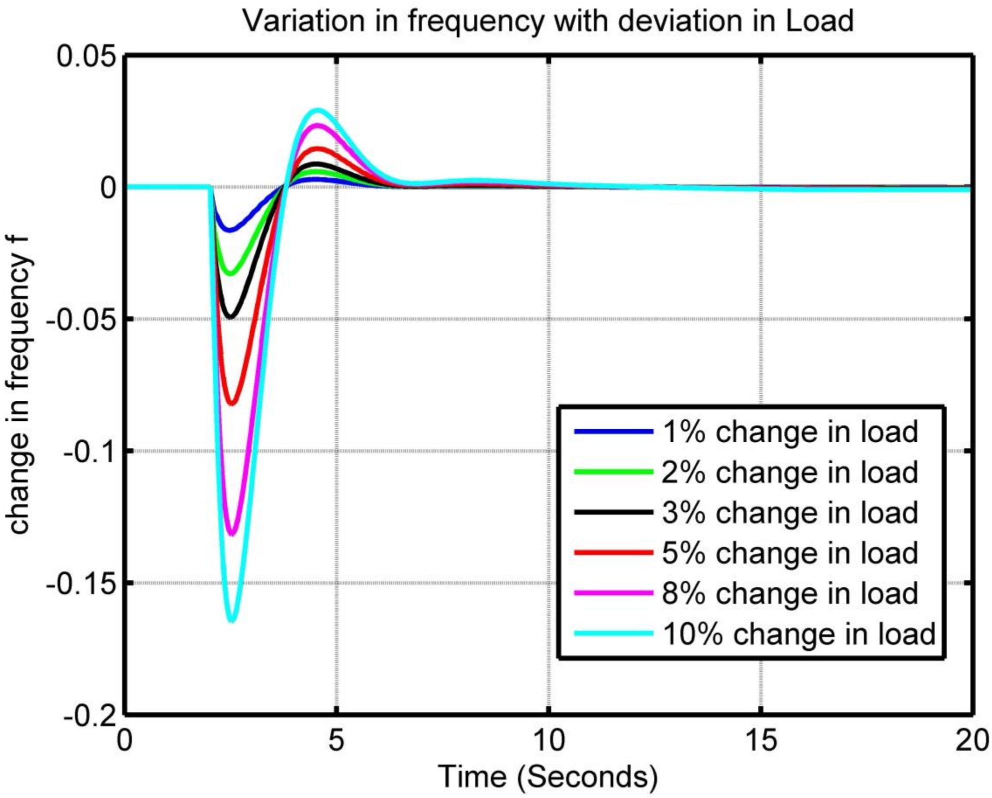


Figure 5.50. Change in frequency with different variations in load at 2 seconds for single area reheated hydrothermal power system

The comparative analysis of the proposed controller is performed with the controller presented in the literature (J. Singh et al., 2018). Change in frequency due to load variation for the single area reheated hydrothermal power system w.r.t time is plotted in figure 5.51. The figure shows the change in frequency after adding the controller derived by implemented technique and Singh technique in the original uncompensated system. Also, the figure reveals that the controller designed by the method proposed in the thesis provides a better value of response than the Singh controller. The 5% change in load occurs at time t=4 seconds, and due to load change, undershoot offered by the proposed controller is 0.82, which is less than 0.89 as possessed by the Singh controller. Moreover, the time taken by the proposed controller to diminish the effect of load variation is 4 seconds, while the Singh controller takes almost 5 seconds to stabilize the frequency variation.

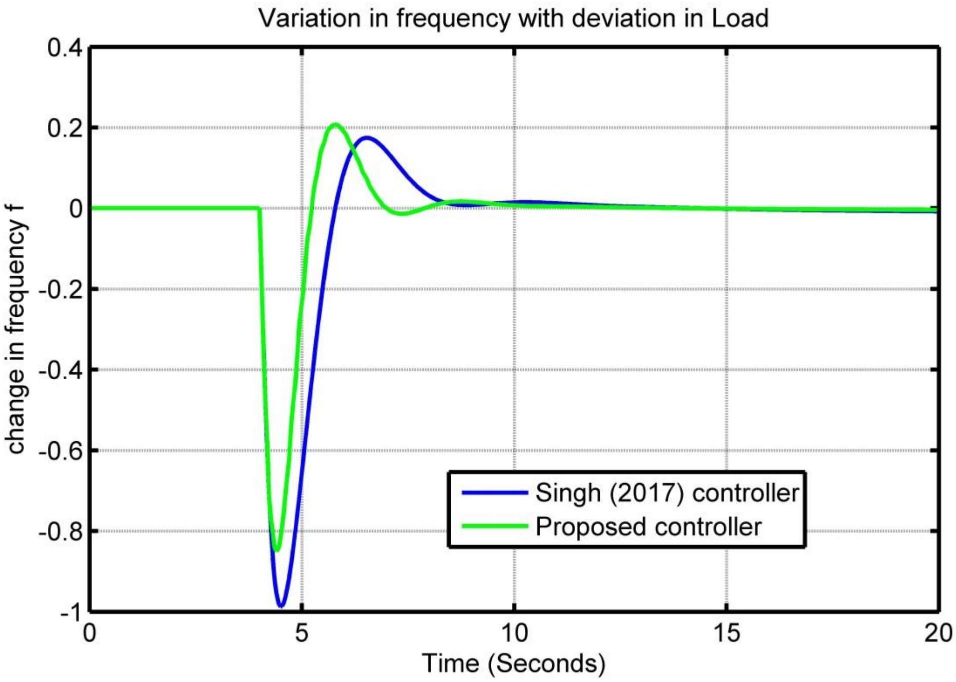


Figure 5.51. Comparison of LFC offered by proposed methodology with the LFC presented in literature when there is a deviation in load at t=4 seconds for single area reheated thermal-hydropower system

Also, the oscillations in both the controllers are minor, providing a good response. So, the comparison reveals that the controller proposed performs far better than another one by providing less undershoot and time utilised in settling the response. Hence, the controller proposed is better for reducing load variation on the frequency of operation and, hence, stabilizing the frequency faster, avoiding any destruction in the power generation system.

Example 2: A power system consisting of two control areas of thermal-thermal non-reheated type, which is to be compensated for the changes in load. Its block diagram in figure 5.52 shows the power system. The diagrammatic view shows two symmetric power generation systems; both are the power system of the non-reheated thermal type having a governor and a non-reheated turbine. These two power systems are interconnected by a power line known as a tie-line.

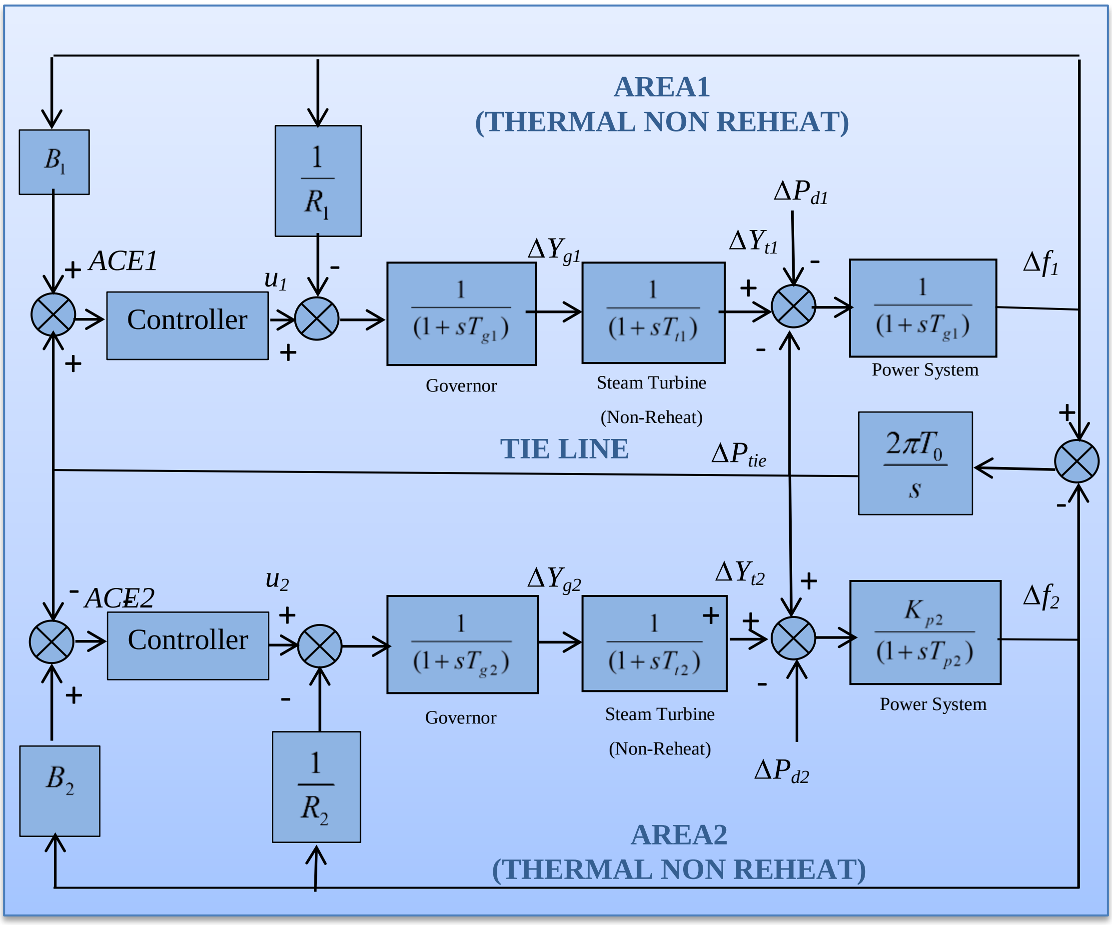


Figure 5.52. Block diagram of the power system of the thermal-thermal non-reheated type having two control areas

Table 5.37 depicts the value of all parameters of the given system having three control areas. The effect of load deviation on the operating frequency of the given system is highlighted in figure 5.53. The figure shows that after load variation, the frequency of operation is changed, and a 0.97 frequency deviation from the normal range is analysed. That means, if we consider that the operating frequency is 50 Hz, then after load variation, it becomes 49.03Hz.

Table 5.37. Nomenclature and parameter values of the power system of the thermal-thermal non-reheated type having two control areas

Symbol	Nomenclature	Value/ Unit
ΔY_{g1}, ΔY_{g2}	The output power of the governor	MW
ΔY_{t1}, ΔY_{t2}	The output power of the turbine	MW
ΔP_{d1}, ΔP_{d2}	Load disturbance	-
ACE1, ACE2	Area Control Error	-
T_{g1}, T_{g2}	The time constant of the thermal generator	0.08 seconds
T_{t1}, T_{t2}	The time constant of the thermal turbine	0.4 seconds
K_{p1}, K_{p2}	Gain of power system	120 Hz/puMW
T_{p1}, T_{p2}	Time constant of power system	20 seconds
R_1, R_2	Regulation in speed of governors	2.4 Hz/pu MW
B_1, B_2	Tie-line frequency bias	0.425 pu MW/Hz
T_0	The synchronizing coefficient for Tie line	0.0707 MW/radian
ΔP_{tie}	Deviation in power at tie line	MW
Δf_1, Δf_2	Frequency deviation in two areas	Hz

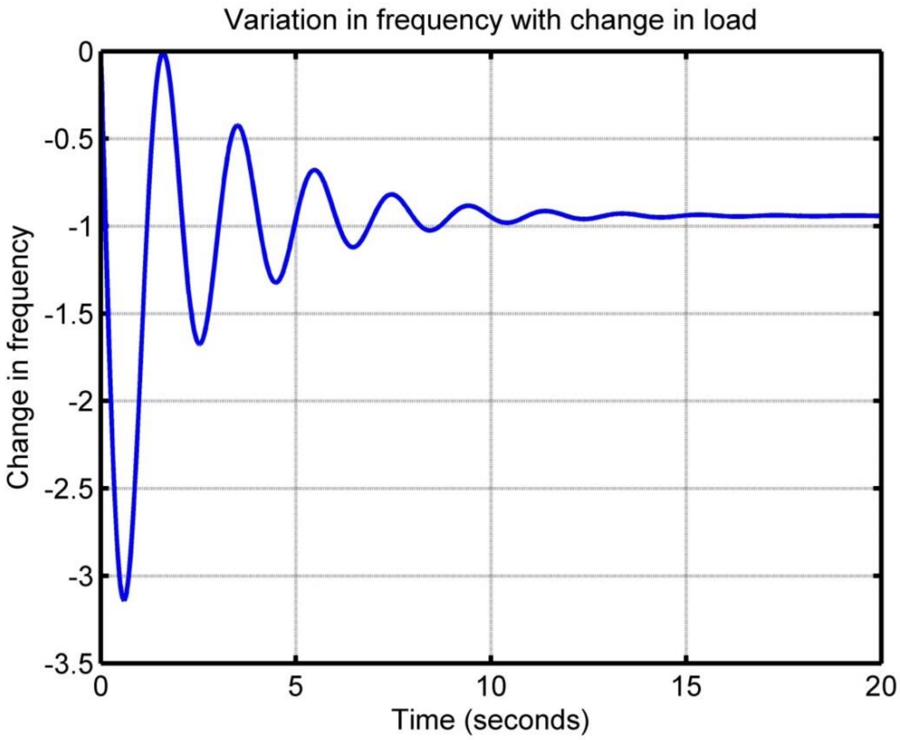


Figure 5.53. Frequency change with variation in load for the uncompensated power system of the thermal-thermal non-reheated type having two control areas

This frequency deviation persists in the system, as is clear from figure 5.53. This change in frequency can damage the devices attached to the power generation unit. So, it necessitates removing the effect of this load variation, which is achieved by implementing a controller in the feedback of the given system.

Figure 5.52 hence helps to procure the transfer function of the system generating power by putting the value of all parameters from Table 5.37. The transfer function is shown in equation (5.61) w.r.t. applied input and equation (5.62) w.r.t. the load fluctuations occurring in the power system.

$$G_i(s) = \begin{bmatrix} \frac{ACE_1(s)}{\Delta R_1(s)} & \frac{ACE_1(s)}{\Delta R_2(s)} \\ \frac{ACE_2(s)}{\Delta R_1(s)} & \frac{ACE_2(s)}{\Delta R_2(s)} \end{bmatrix} = \frac{1}{D_7(s)} \begin{bmatrix} n_{11}(s) & n_{12}(s) \\ n_{21}(s) & n_{22}(s) \end{bmatrix}, \text{ and} \quad (5.61)$$

$$G_d(s) = \begin{bmatrix} \frac{ACE_1(s)}{\Delta P_{D1}(s)} & \frac{ACE_1(s)}{\Delta P_{D1}(s)} \\ \frac{ACE_2(s)}{\Delta P_{D2}(s)} & \frac{ACE_2(s)}{\Delta P_{D2}(s)} \end{bmatrix} = \frac{1}{D_7(s)} \begin{bmatrix} n_{11}{}^l(s) & n_{12}{}^l(s) \\ n_{21}{}^l(s) & n_{22}{}^l(s) \end{bmatrix} \quad (5.62)$$

The equation of denominator is the same in both the transfer functions and is shown in equation (5.63) as follows:

$$\begin{aligned} D_7(s) &= s^7 + 30.1s^6 + 295.8329s^5 + 1282.7525s^4 + 4963.0643s^3 \\ &+ 10590.277s^2 + 18051.8717s + 13273.862 \end{aligned} \quad (5.63)$$

Moreover, the numerator polynomial for the input *[R(s)]* is shown in equation (5.64) as follows:

$$n_{11}(s) = n_{22}(s) = 79.6875s^4 + 1282.58437s^3 + 4015.86s^2 + 12201.0047s + 13273.90367$$

$$n_{12}(s) = n_{21}(s) = -83.2875s^3 - 1041.09375s^2 + 520.546875s + 0.0416437$$

(5.64)

And, the numerator polynomial for the load disturbance as input *[$P_D(s)$]* is given by equation (5.65) as follows:

$$\begin{aligned} n_{11}{}^l(s) = n_{22}{}^l(s) &= -25.5s^6 - 792.927s^5 - 8238.3552s^4 - 36006.2932s^3 - 102971.0717s^2 \\ &- 185724.7845s - 132739.0367 \\ n_{12}{}^l(s) = n_{21}{}^l(s) &= 26.652s^5 + 732.9299s^4 + 5663.5499s^3 + 7912.2992s^2 \\ &- 5205.6686s - 0.41644 \end{aligned} \quad (5.65)$$

By adding both transfer functions as shown in equation (5.66) for area 1 and equation (5.57) for area 2, the combined transfer function of the given system for both inputs (applied input and load fluctuation) is procured. As both areas of the power system are symmetrical, the combined transfer function for both areas is the same and derived as shown in equation (5.68).

$$G_{71}(s) = \frac{1}{D_7(s)}\left[(n_{11}(s) + n_{11}{}^{l}(s)) + (n_{12}(s) + n_{12}{}^{l}(s))\right], \text{ and} \quad (5.66)$$

$$G_{72}(s) = \frac{1}{D_7(s)}\left[(n_{21}(s) + n_{21}{}^{l}(s)) + (n_{22}(s) + n_{22}{}^{l}(s))\right] = G_{71}(s) \quad (5.67)$$

$$G_{71}(s) = G_{72}(s) = \frac{\begin{matrix} -25.5s^6 - 766.2750s^5 - 7425.7383s^4 - 29143.4473s^3 \\ -92084.0078s^2 - 178208.8906s - 119465.5 \end{matrix}}{\begin{matrix} s^7 + 30.1s^6 + 295.8329s^5 + 1282.7525s^4 + 4963.0643s^3 \\ +10590.277s^2 + 18051.8717s + 13273.862 \end{matrix}} \quad (5.68)$$

Hence, the transfer function of the power system of the thermal-thermal non-reheated type having two control areas is procured in equation (5.68). The transfer function is to be reduced in the 3rd order approximation from the technique developed in the research. As the original system is of 7th order, so the numbers of poles of the system are seven, which are obtained as: *σ_1=-13.0677, σ_2 = -13.0522, σ_3= -1.2376, σ_4, σ_5= -0.9911±2.2618i and σ_6, σ_7= -0.3802±3.1887i*. By applying the implemented technique of MOR, the poles of the system of reduced-order are procured as, *σ_{c1}= -1.2537* and *σ_{c2}, σ_{c3}= -0.4597±2.5722i*. The transfer function of the system of reduced-order is obtained using these poles and performing optimisation. After 48 iterations, the transfer function for the system of the reduced order is procured and highlighted by equation (5.69).

$$G_3(s) = \frac{-15.7s^2 - 63.2s - 77.7}{s^3 + 2.171s^2 + 7.9765s + 8.558} \quad (5.69)$$

Figure 5.54 depicts the response of the original system of high order and the system of reduced-order in the time-domain when the unit step signal is applied at the input. It is revealed from the figure that the response of both systems is almost the same. Both responses start from the same value and terminate at the same value showing zero value of error at a steady state. The system of reduced-order provides some extra overshoot in comparison to the system of higher-order. Furthermore, the value of time-domain performance parameters in both systems is procured in Table 5.38. The table describes that most of the characteristics related to the time-domain have the same value for both systems. The value of rise time is almost similar, but the response of a reduced-order system takes more time to settle down in comparison to the system of higher-order.

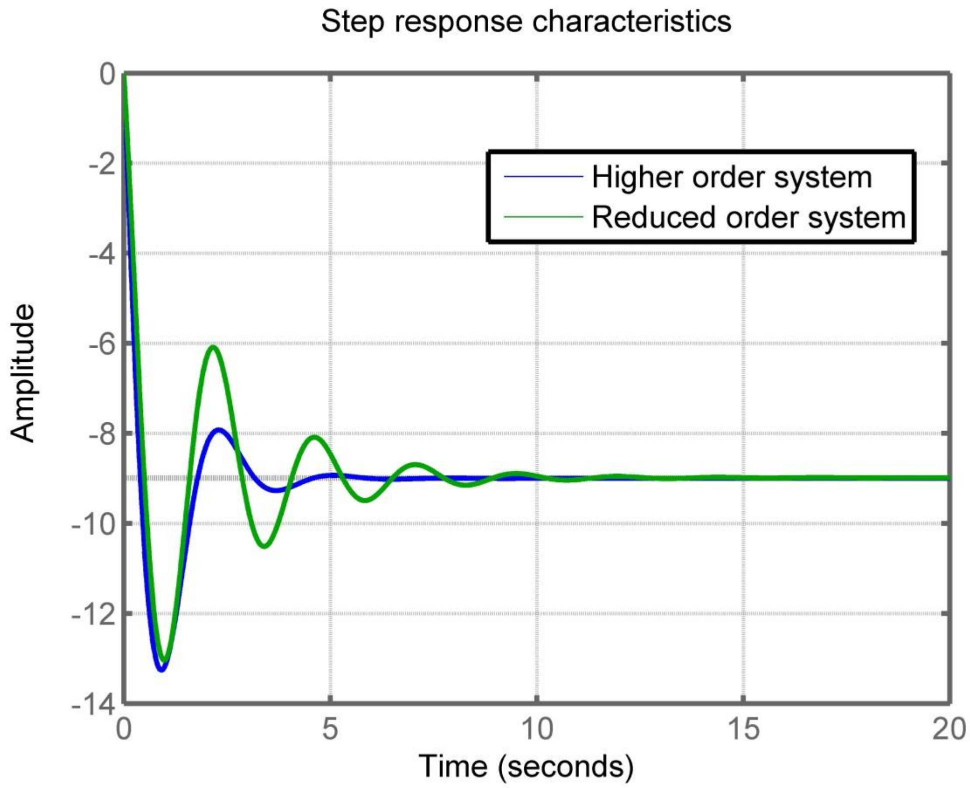


Figure 5.54. Step response characteristics of 7th order and reduced 3rd order of thermal-thermal non-reheated power system having two control areas as obtained from the implemented technique

Table 5.38. Value of errors and various parameters to compare given high order power system of the thermal-thermal non-reheated type with two control areas and obtained 3rd order system of example 2 for the unit step input

Parameter	ISE	ITAE	IAE	OS (%)	RT (sec.)	ST (sec.)	SSE
HOS	-	-	-	47.2840	0.3133	4.0761	-
ROS	0.5608	1.6692	0.5344	45.0310	0.3871	7.4245	0

Moreover, the value of overshoot is almost similar in both systems, with zero value of error at a steady state. Furthermore, the value of ISE, ITAE and IAE reveal that the value of error in the system of a reduced and high order is significantly less. Hence, the reduced-order approximation obtained from the implemented MOR technique is a sound approximated system of the given power system. So the procured system of reduced-order is suitable to be used instead of the system of higher-order for designing the compensator for the power system of the thermal-thermal non-reheated type having two control areas.

Hence the design of the 2-DOF controller of IMC-PID type for the two area system has consisted of the following steps as described in methodology:

1. *Factorization:* Initially, the factorization of the reduced-order system in two parts (minimal and non-minimal) is performed, which results in equation (5.70) as follows:

$$G_3(s) = G_3^-(s)\cdot G_3^+(s) = \left[\frac{-15.7s^2 - 63.2s - 77.7}{s^3 + 2.171s^2 + 7.9765s + 8.558}\right]\cdot 1 \qquad (5.70)$$

2. *Inversion:* Then, the inversion of minimal part in equation (5.70), keeping the non-minimal part intact, results in the compensator equation (5.71) for two areas thermal-thermal non-reheated system as follows:

$$Q_{c1}(s) = \left[G_3^-(s)\right]^{-1}\cdot G_3^+(s) = \left[\frac{-15.7s^2 - 63.2s - 77.7}{s^3 + 2.171s^2 + 7.9765s + 8.558}\right]^{-1}\cdot 1 \qquad (5.71)$$

3. *Filtration:* The compensator obtained in equation (5.71) is not proper, so a low pass filter of degree 1 is added in the cascade of the compensator. So, after adding the low pass filter, the equation for compensator becomes:

$$Q_{c1}(s) = \left[G_3^-(s)\right]^{-1}\cdot G_3^{+}(s)\cdot F(s) = \left(\frac{-15.7s^2 - 63.2s - 77.7}{s^3 + 2.171s^2 + 7.9765s + 8.558}\right)^{-1}\cdot\left(\frac{1}{\lambda s + 1}\right) \qquad (5.72)$$

For obtaining the optimal IMC compensator, the equation (5.72) is optimised to find the value of filter time constant *'λ'*. The value of *'λ'* is obtained from the optimisation of the equation (5.72), which comes out to be 0.0636 after 29 iterations. By putting the value of *'λ'* in equation (5.72), the optimised IMC compensator can be written as follows:

$$Q_{c1}(s) = \frac{s^3 + 2.171s^2 + 7.9765s + 8.558}{-0.9985s^3 - 19.7195s^2 - 68.1417s - 77.7} \quad (5.73)$$

4. For designing the disturbance rejecting compensator of equation (4.25) for removing the effect of load disturbance, the value of *'m'* is taken equal to one, and the value of λ_2 is equal to 0.3. Computing the value of k_1 by putting the value of 'm' and 'λ_2' results in 0.3626. This value designs the disturbance rejecting compensator, which is represented by equation (5.74).

$$Q_{c2}(s) = \frac{1 + 0.3626s}{1 + 0.3s} \quad (5.74)$$

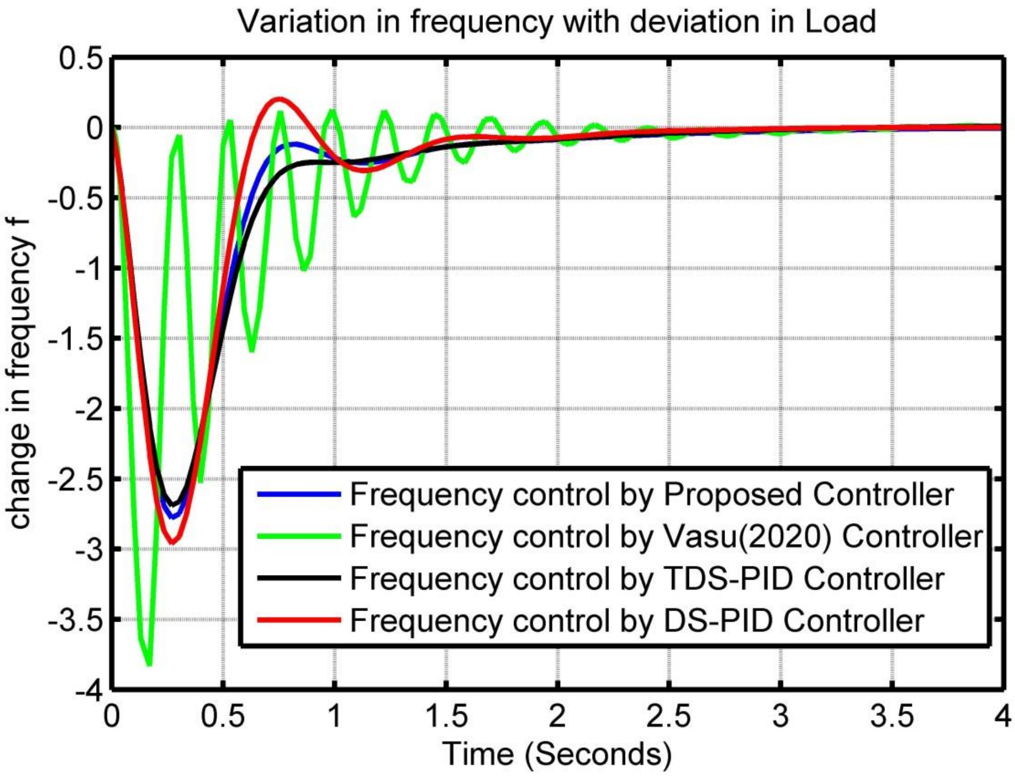


Figure 5.55. Comparison of LFC offered by proposed methodology with the LFC presented in the literature for the power system of the thermal-thermal non-reheated type having two control areas

The 2-DOF compensator combines the IMC-PID compensator of equation (5.73) and the disturbance rejecting compensator of equation (5.74). These two compensators are combined to form the IMC-PID controller. After combining both compensators, the IMC-PID feedback controller is obtained and is written as:

$$C(s)=\frac{Q_{c1}(s)\cdot Q_{c2}(s)}{1-G_3(s)\cdot Q_{c1}(s).Q_{c2}(s)}=\frac{s^4+4.931s^3+13.97s^2+30.57s+23.62}{0.8268s^4+3.385s^3+4.319s^2+0.2787s} \quad (5.75)$$

By comparing the IMC controller obtained in equation (5.75) with the ideal PID controller by a least-square model matching method as shown in equation (5.30), the gain parameters of the 2-DOF IMC-PID controller are obtained as:

$K_p = 2.0609, K_i = 2.0766, K_d = 1.1614$ (5.76)

By using the gain parameters obtained in equation (5.76), the variation in frequency due to load disturbance is analysed, which is shown in figure 5.55. The figure shows an initial variation in frequency due to a 40% variation in load. But after almost 3 seconds, the variation in frequency diminishes. The implemented system possesses a lesser number of oscillations and a low value of undershooting. A comparative analysis is performed amongst the frequency variation by implementing the newly designed controller and the controller developed in literature (Anwar & Pan, 2015; Sahu et al., 2016; Vasu et al., 2021). The comparative analysis performed in figure 5.55 shows that the deviation in the Vasu controller shows oscillatory behaviour, and these fluctuations can be sometimes dangerous and can cause damage to the system if load variation comes in between the operation.

On the other hand, implemented controller does not exhibit oscillations, hence making the implemented controller more valuable than Vasu. The amount of undershoot exhibited by the proposed controller is *-2.72,* while Vasu's controller exhibits undershoot equal to *-3.79.* The DS-PID and TDS-PID controller exhibit undershoot of *-2.94* and *-2.71,* respectively. Hence based on the value of undershoot, the implemented controller and TDS-PID controller is termed as better than the Vasu controller and DS-PID controller. All the parameters like undershoot and number of oscillations in implemented and TDS-PID controller are almost similar. Moreover, the settling time of all controllers is almost the same, which is approximately 3 seconds. After 3 seconds, the deviation in frequency gets eliminated and hence neglects the effect of load deviation, which is the primary purpose of designing an LFC controller. So, the effect of load variation vanishes after 3 seconds, and the system becomes stable, and the working of the power system becomes normal. The significant parameters to check the fluctuations in frequency deviation are settling time, overshoot/ undershoot and oscillations. The implemented controller fits all these parameters, and hence it is feasible to use the implemented controller to reduce the effect of load perturbation.

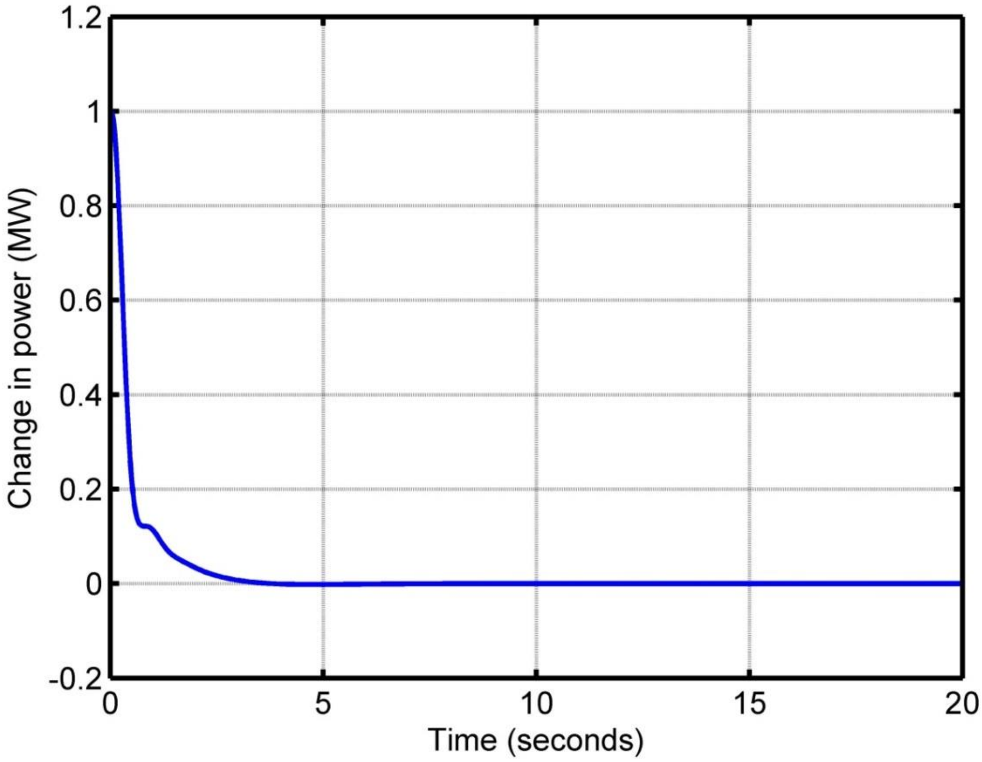


Figure 5.56. Power deviation in the power system of the thermal-thermal non-reheated type with two control areas after adding the implemented controller

Moreover, figure 5.56 shows that the effect of change in load on tie-line power also gets reduced to zero after 3 seconds. Initially, after experiencing a change in load, the power of the tie-line changes to 1 MW, but after that, it starts decreasing and finally becomes zero after a short period. It signifies that the controller proposed here reduces the effect of load perturbation on the frequency of operation and tie-line power in a more efficient way than previously designed controllers.

Example 3: An interconnected power system with three control areas with solar cell (Photovoltaic) as one area and other two areas are thermal-thermal power systems considered for controller designing. The three power systems are interconnected together to form a combined power system. The three different power systems are termed as three areas of operation. The diagrammatic form of the power system of PV-thermal-thermal type having three control areas with 2-DOF IMC-PID controller is drawn in figure 5.57. The first area of the power system is an on-grid solar power generating unit connected with the non-reheated thermal power generating unit of the second and third areas. The complete solar PV power generator is consists of a solar cell (PV cell), DC-DC boost convertor, maximum power point tracking (MPPT) to get fixed output at each instant of time, invertor (to get ac output) and a low pass filter (Tomy, 2014). The combined transfer function *'$G_{PV}(s)$'* of the solar power generation area is hence obtained and is given by equation (5.77)

$$G_{PV}(s) = \frac{-18s + 900}{s^2 + 100s + 50} \tag{5.77}$$

The second area is symmetrical to the third area of the power system. It is a thermal non-reheated power system having a governor with transfer function *'$G_g(s)$'* with a time constant *'T_g'* as shown by equation (5.78) and a turbine with transfer function *'$G_t(s)$'* with a time constant *'T_t'* as shown by equation (5.79). The power system's transfer function *'$G_{ps}(s)$'* is formed of time constant *'T_p'* and gain of power system is *'$K_p(s)$'* written as shown by equation (5.80).

$$G_g(s) = \frac{1}{1 + T_g.s} \tag{5.78}$$

$$G_t(s) = \frac{1}{1 + T_t.s} \tag{5.79}$$

$$G_{ps}(s) = \frac{K_p}{1 + T_p.s} \tag{5.80}$$

The transfer function *'$G_{tl}(s)$'* of tie power line depends on the synchronizing coefficient for tie-line of two areas *'T_{tl}'* and is written by equation (5.81).

$$G_{tl}(s) = \frac{2\prod T_{tl}}{s} \tag{5.81}$$

Table 5.39 reveals the value of the parameters of the given power generating system consisted of three areas. The change in frequency due to variation in load on the power system can be analysed in figure 5.58 for area 1 and figure 5.59 for areas 2 and 3. Figure 5.58 reveals that due to the 10% variation in load at 30 seconds, there is a deviation in the operating frequency of the PV solar system to a value of 0.1487. It means that after applying a load on the power system, the operating frequency becomes 50.1487 Hz instead of 50 Hz. (50Hz is the standard frequency range).

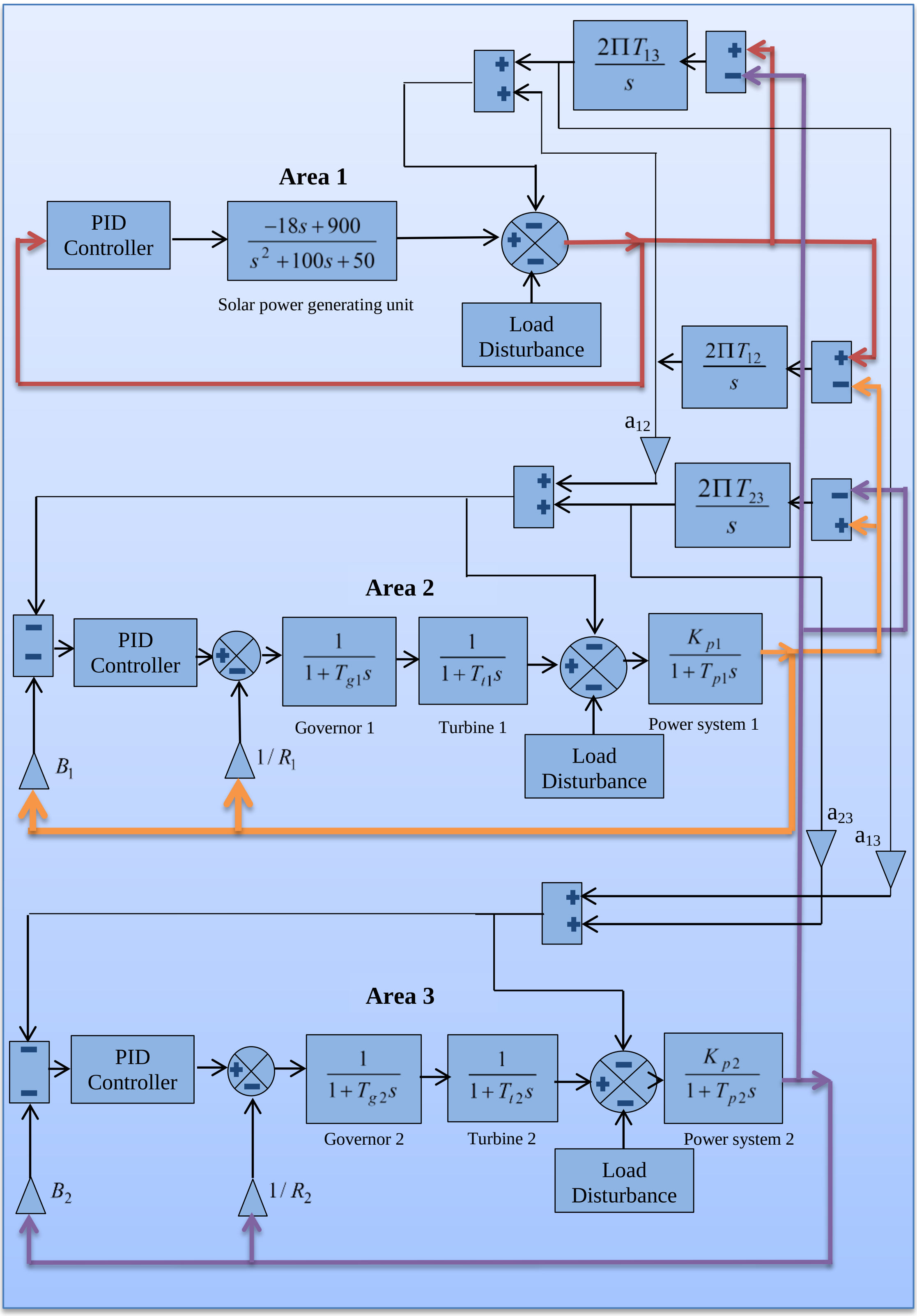


Figure 5.57. Power system's block diagram of PV-thermal-thermal type consisting of three control areas

Table 5.39. Parameter values of the power system of PV-thermal-thermal power type having three control areas

Parameter	Value
T_{t1}, T_{t2}	0.3 sec
T_{g1}, T_{g2}	0.08 sec
K_{p1}, K_{p2}	100
T_{p1}, T_{p2}	20 sec
R_1, R_2	2 Hz/pu MW
a_{12}, a_{23}, a_{31}	-1
T_{12}, T_{23}, T_{13}	0.0707
B_1, B_2	0.425 pu MW/Hz

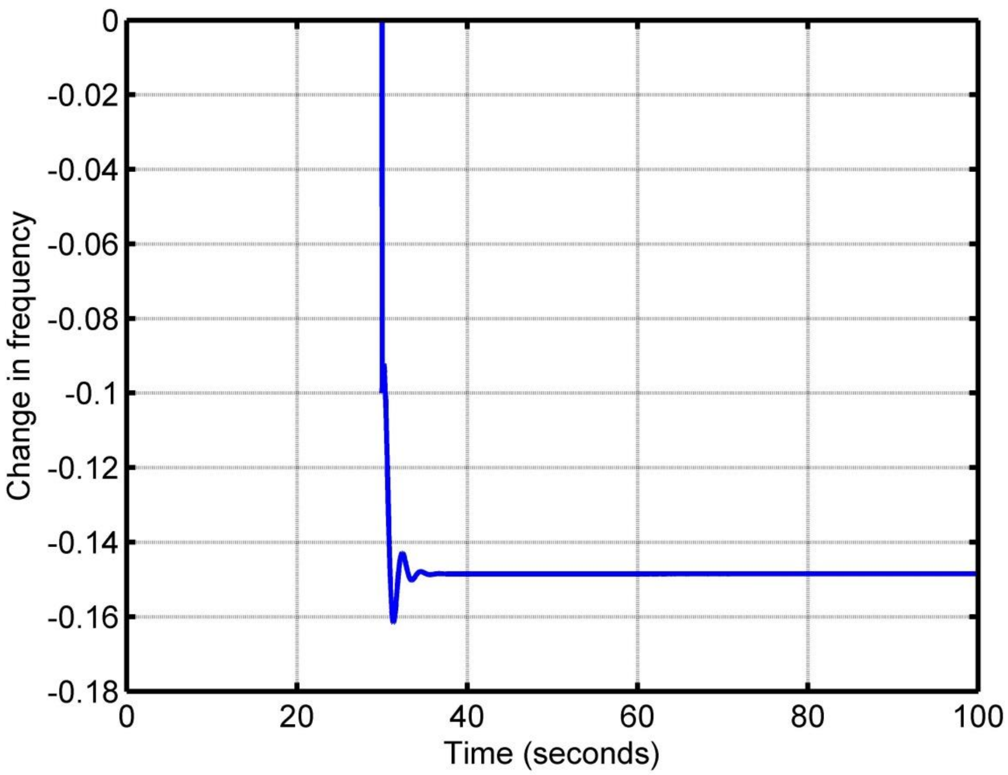


Figure 5.58. Change in frequency with variation in load for area 1 of the uncompensated power system

Similarly, figure 5.59 depicts that the frequency of operation deviates to a value of 0.1475 in areas two and three. So it is clear that the frequency of operation changes because of variation in load or applying a load on the power system. From the discussion, it is clear that the load variation causes changes in the frequency of the power generation system, which is termed the LFC problem and hence the problem of LFC is to be removed. The best possible way to cope with the load frequency problem is to add a controller in the feedback of each control area.

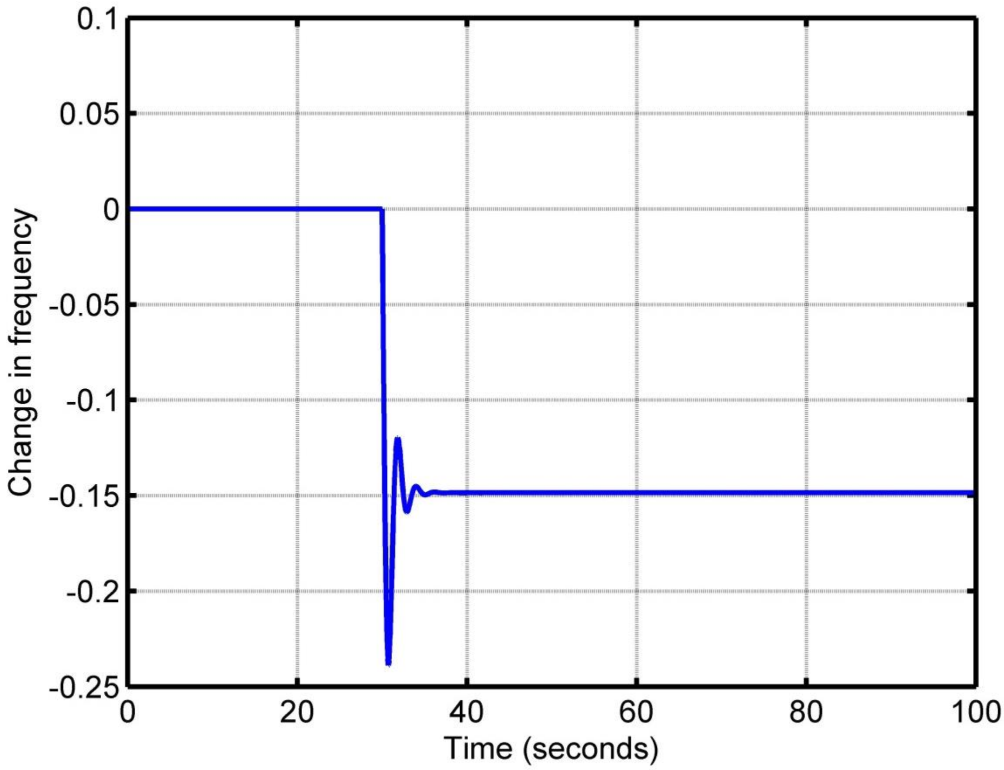


Figure 5.59. Change in frequency with variation in load for areas 2 & 3 of the uncompensated power system

So the design of a suitable controller is required for all the three control areas of the power system formed from the integration of PV generation system and thermal-thermal systems given in figure 5.57 so that the effect of load variation can be eliminated in the power system and change in frequency and power can be neutralized. Hence, the frequency deviation experienced in figures 5.58 and figure 5.59 is to be removed using the implemented controller.

The process procured in designing the controller of IMC-PID type executes in the steps as described in the methodology section. The outcome of the application of these steps is given as:

1. It is depicted from the figure 5.57 that the power system consists of three areas, which signifies that there are three outputs of the power system (One for each system) and each area has its corresponding input and load disturbance. Considering

that the inputs are represented by u_1, u_2, u_3, three outputs are denoted by the symbols C_1, C_2, C_3 and the load disturbance entering in each area is represented as D_1, D_2, D_3 for area 1, area 2 and area 3 respectively. The transfer function associated with each input-output combination for getting output of area 1 is obtained as shown by equations (5.82a-5.82d) described as:

$$\frac{C_1}{R_1}=\frac{\begin{gathered}-4.147s^9+75.62s^8+5152s^7+6.412\times10^4s^6+3.483\times10^5s^5+1.603\times10^6s^4\\+4.427\times10^6s^3+9.48\times10^6s^2+1.21\times10^7s+5.327\times10^6\end{gathered}}{\begin{gathered}0.00553s^{12}+0.8211s^{11}+32.25s^{10}+581.5s^9+5803s^8+3.694\times10^4s^7\\+1.723\times10^5s^6+6.004\times10^5s^5+1.548\times10^6s^4+2.931\times10^6s^3+3.658\times10^6s^2\\+2.45\times10^6s+5.973\times10^5\end{gathered}} \tag{5.82a}$$

$$\frac{C_1}{R_2}=\frac{C_1}{R_3}=\frac{\begin{gathered}21.32s^6+2471s^5+3.598\times10^4s^4+1.262\times10^5s^3+5.098\times10^5s^2\\+8.177\times10^5s+2.96\times10^5\end{gathered}}{\begin{gathered}0.00553s^{12}+0.8211s^{11}+32.25s^{10}+581.5s^9+5803s^8+3.694\times10^4s^7\\+1.723\times10^5s^6+6.004\times10^5s^5+1.548\times10^6s^4+2.931\times10^6s^3\\+3.658\times10^6s^2+2.45\times10^6s+5.973\times10^5\end{gathered}} \tag{5.82b}$$

$$\frac{C_1}{D_1}=\frac{\begin{gathered}-0.2304s^{10}-30.36s^9-823.2s^8-8764s^7-4.833\times10^4s^6-2.147\times10^5s^5\\-6.185\times10^5s^4-1.351\times10^6s^3-1.902\times10^6s^2-1.27\times10^6s-2.96\times10^5\end{gathered}}{\begin{gathered}0.00553s^{12}+0.8211s^{11}+32.25s^{10}+581.5s^9+5803s^8+3.694\times10^4s^7\\+1.723\times10^5s^6+6.004\times10^5s^5+1.548\times10^6s^4+2.931\times10^6s^3+3.658\times10^6s^2\\+2.45\times10^6s+5.973\times10^5\end{gathered}} \tag{5.82c}$$

$$\frac{C_1}{D_2}=\frac{C_1}{D_3}=\frac{\begin{gathered}-0.5117s^8-67.4s^7-1824s^6-1.917\times10^4s^5-9.616\times10^4s^4\\-3.395\times10^5s^3-8.276\times10^5s^2-9.301\times10^5s-2.96\times10^5\end{gathered}}{\begin{gathered}0.00553s^{12}+0.8211s^{11}+32.25s^{10}+581.5s^9+5803s^8+3.694\times10^4s^7\\+1.723\times10^5s^6+6.004\times10^5s^5+1.548\times10^6s^4+2.931\times10^6s^3\\+3.658\times10^6s^2+2.45\times10^6s+5.973\times10^5\end{gathered}} \tag{5.82d}$$

The equation C_1/R_2 is equal to the equation C_1/R_3, and the equation C_1/D_2 is equal to the equation C_1/D_3 due to the symmetry of second and third areas.

From equation (5.82a-5.82d), the complete transfer function related to the first output is formed by combining the transfer function corresponding to each input and transfer functions corresponding to each load disturbance input. So the overall transfer function related to the first output is obtained by adding the input and disturbance transfer functions $G_1(s) = (C_1/R_1 + C_1/R_2 + C_1/R_3) + (C_1/D_1 + C_1/D_2 + C_1/D_3)$. The result of this addition is displayed in equation (5.83).

$$G_1(s) = \frac{\begin{array}{c} -0.2304s^{10} - 34.51s^9 - 747.6s^8 - 3613s^7 + 1.577\times10^4 s^6 + 1.334\times10^5 s^5 \\ + 9.833\times10^5 s^4 + 3.073\times10^6 s^3 + 7.57\times10^6 s^2 + 1.082\times10^7 s + 5.027\times10^6 \end{array}}{\begin{array}{c} 0.00553s^{12} + 0.8211s^{11} + 32.25s^{10} + 581.5s^9 + 5803s^8 + 3.694\times10^4 s^7 \\ + 1.723\times10^5 s^6 + 6.004\times10^5 s^5 + 1.548\times10^6 s^4 + 2.931\times10^6 s^3 + 3.658\times10^6 s^2 \\ + 2.45\times10^6 s + 5.973\times10^5 \end{array}} \quad (5.83)$$

The equation (5.83) represents the combined transfer function *'$G_1(s)$'* related to the first output or output of area 1, which is used further to obtain the IMC-PID controller for area 1. Similarly, the transfer functions for the output of area 2 and area 3 are represented by equations (5.84a-5.84c) for applied input and equation (5.84d-5.84f) for load disturbances.

$$\frac{C_2}{R_1} = \frac{C_3}{R_1} = \frac{\begin{array}{c} -9.211s^7 + 168.4s^6 + 1.145\times10^4 s^5 + 1.424\times10^5 s^4 + 7.082\times10^5 s^3 \\ + 2.493\times10^6 s^2 + 5.981\times10^6 s + 5.327\times10^6 \end{array}}{\begin{array}{c} 0.00553s^{12} + 0.8211s^{11} + 32.25s^{10} + 581.5s^9 + 5803s^8 + 3.694\times10^4 s^7 \\ + 1.723\times10^5 s^6 + 6.004\times10^5 s^5 + 1.548\times10^6 s^4 + 2.931\times10^6 s^3 \\ + 3.658\times10^6 s^2 + 2.45\times10^6 s + 5.973\times10^5 \end{array}} \quad (5.84a)$$

$$\frac{C_2}{R_2} = \frac{C_3}{R_3} = \frac{\begin{array}{c} 48s^7 + 5605s^6 + 8.583\times10^4 s^5 + 3.435\times10^5 s^4 + 1.205\times10^6 s^3 \\ + 2.094\times10^6 s^2 + 1.375\times10^6 s + 2.96\times10^5 \end{array}}{\begin{array}{c} 0.00553s^{12} + 0.8211s^{11} + 32.25s^{10} + 581.5s^9 + 5803s^8 + 3.694\times10^4 s^7 \\ + 1.723\times10^5 s^6 + 6.004\times10^5 s^5 + 1.548\times10^6 s^4 + 2.931\times10^6 s^3 \\ + 3.658\times10^6 s^2 + 2.45\times10^6 s + 5.973\times10^5 \end{array}} \quad (5.84b)$$

$$\frac{C_2}{R_3}=\frac{C_3}{R_2}=\frac{\begin{array}{c}116.1s^5+1.354\times10^4s^4+2.055\times10^5s^3+7.35\times10^5s^2\\+7.138\times10^5s+1.983\times10^5\end{array}}{\begin{array}{c}0.00553s^{12}+0.8211s^{11}+32.25s^{10}+581.5s^9+5803s^8+3.694\times10^4s^7\\+1.723\times10^5s^6+6.004\times10^5s^5+1.548\times10^6s^4+2.931\times10^6s^3\\+3.658\times10^6s^2+2.45\times10^6s+5.973\times10^5\end{array}}$$

(5.84c)

$$\frac{C_2}{D_1}=\frac{C_3}{D_1}=\frac{\begin{array}{c}-0.5117s^8-67.4s^7-1824s^6-1.917\times10^4s^5-9.616\times10^4s^4\\-3.395\times10^5s^3-8.276\times10^5s^2-9.301\times10^5s-2.96\times10^5\end{array}}{\begin{array}{c}0.00553s^{12}+0.8211s^{11}+32.25s^{10}+581.5s^9+5803s^8+3.694\times10^4s^7\\+1.723\times10^5s^6+6.004\times10^5s^5+1.548\times10^6s^4+2.931\times10^6s^3\\+3.658\times10^6s^2+2.45\times10^6s+5.973\times10^5\end{array}}$$

(5.84d)

$$\frac{C_2}{D_2}=\frac{C_3}{D_3}=\frac{\begin{array}{c}-1.152s^9-152.8s^8-4238s^7-4.647\times10^4s^6-2.453\times10^5s^5\\-8.517\times10^5s^4-2.034\times10^6s^3-2.623\times10^6s^2-1.488\times10^6s-2.96\times10^5\end{array}}{\begin{array}{c}0.00553s^{12}+0.8211s^{11}+32.25s^{10}+581.5s^9+5803s^8+3.694\times10^4s^7\\+1.723\times10^5s^6+6.004\times10^5s^5+1.548\times10^6s^4+2.931\times10^6s^3\\+3.658\times10^6s^2+2.45\times10^6s+5.973\times10^5\end{array}}$$

(5.84e)

$$\frac{C_2}{D_3}=\frac{C_3}{D_2}=\frac{\begin{array}{c}-2.786s^7-369.1s^6-1.019\times10^4s^5-1.093\times10^5s^4-5.019\times10^5s^3\\-1.011\times10^6s^2-7.891\times10^5s-1.983\times10^5\end{array}}{\begin{array}{c}0.00553s^{12}+0.8211s^{11}+32.25s^{10}+581.5s^9+5803s^8+3.694\times10^4s^7\\+1.723\times10^5s^6+6.004\times10^5s^5+1.548\times10^6s^4+2.931\times10^6s^3\\+3.658\times10^6s^2+2.45\times10^6s+5.973\times10^5\end{array}}$$

(5.84f)

The equalities in the transfer functions seen in the equations (5.84a-5.84f), result from the symmetrical nature of areas 2 and 3. After adding all the transfer functions (containing applied input and load disturbance input) related to area 2 output as $G_2(s)=(C_2/R_1+C_2/R_2+C_2/R_3)+(C_2/D_1+C_2/D_2+C_2/D_3)$ and transfer functions related to the output of area 3 output as $G_3(s)=(C_3/R_1+C_3/R_2+C_3/R_3)+(C_3/D_1+C_3/D_2+C_3/D_3)$, the total transfer function for the area 2 *'$G_2(s)$'* and for area 3 *'$G_3(s)$'* is obtained and written in equation (5.85). As area 2 and area 3 are symmetrical and have the same input-output combinations, so the equation for the total transfer function will be similar for both areas. Hence $G_2(s) = G_3(s)$.

$$G_2(s) = G_3(s) = \frac{-0.02765s^9 - 1.851s^8 - 101.1s^7 - 1871s^6 - 8740s^5 + 4.656\times10^4 s^4 + 3.914\times10^5 s^3 + 1.401\times10^6 s^2 + 3.994\times10^6 s + 5.027\times10^6}{0.00553s^{12} + 0.8211s^{11} + 32.25s^{10} + 581.5s^9 + 5803s^8 + 3.694\times10^4 s^7 + 1.723\times10^5 s^6 + 6.004\times10^5 s^5 + 1.548\times10^6 s^4 + 2.931\times10^6 s^3 + 3.658\times10^6 s^2 + 2.45\times10^6 s + 5.973\times10^5} \tag{5.85}$$

The total transfer functions obtained in equations (5.83) & (5.85) are utilised to solve the problem of LFC in the power system having three areas as shown in figure 5.57 by designing a suitable IMC-PID controller.

2. By employing the MOR technique presented in the methodology, the reduced-order transfer functions *'G_{1r}'* for area 1 and *'G_{2r}'* for area 2 and area 3 is obtained as follows:

$$G_{1r}(s) = \frac{2.75s^3 + 0.00019s^2 + 49.6s + 65.3}{s^4 + 3.165s^3 + 13.43s^2 + 21.42s + 7.733} \tag{5.86a}$$

$$G_{2r}(s) = \frac{-3.3s^3 + 8.08s^2 - 32.8s + 65}{s^4 + 3.165s^3 + 13.43s^2 + 21.42s + 7.733} \tag{5.86b}$$

The transfer functions of equations (5.83) & (5.85) are of 12^{th} order, consisting of 12 poles, and these transfer functions are curtailed to the system of lower order having transfer function as shown in equation (5.86a-5.86b) which is of 4^{th} order and hence having four poles. Hence the reduction of the system of 12^{th} order is performed by the IPCG technique to procure the system of 4^{th} order. Figure 5.60 shows the step response behaviour of the higher model shown by equation (5.83) and the reduced model obtained in equation (5.86a) for area 1. The figure exhibits that the path followed by both the systems is the same; hence both systems are said to have similar step response behaviour. Also, Table 5.40 signifies that for area 1, the value of all performance characteristics is almost similar. So it is revealed that the model of reduced-order procured in equation (5.86a) is similar to the higher-order system of equation (5.83). Hence, the procured model of reduced-order is suitable for procuring the controller for area 1.

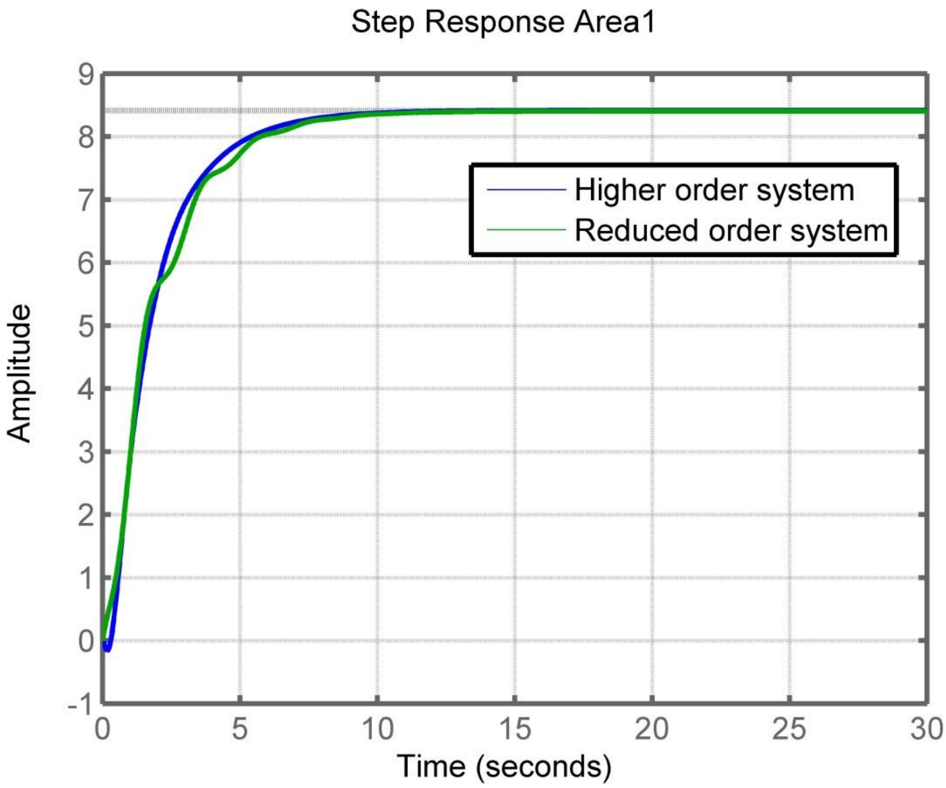


Figure 5.60. Response of the system of high order and reduced order of area 1 of the power system of PV-thermal-thermal having three control areas for the unit step input

Table 5.40. Value of errors and various parameters to compare higher-order power system of PV-thermal-thermal type with three control areas and obtained reduced-order system of example 3 for the unit step input

System	Rise Time	Overshoot	Settling Time	Steady-state value
		Area 1		
HOS	3.5357	0	7.1710	8.4162
ROS	4.2259	0	7.5317	8.4055
		Area 2 & Area 3		
HOS	4.8791	0	9.2931	8.4162
ROS	4.6957	0	9.0504	8.4055

Similarly, figure 5.61 reveals that the response of the system of the high order of equation (5.85) and ROM of equation (5.86b) for unit step input are almost similar, following the same path. Table 5.40 reveals that the performance characteristics of the initial and final state of both systems are almost similar and possess the same stability behaviour. So by analysing figure 5.61 and Table 5.40, it is clear that the ROM of equation (5.86b) is similar to the higher-order model of equation (5.85). Hence, the obtained ROM can be used in a higher-order system of area 2 and area 3 to obtain a controller to remove the LFC problem occurring in the PV generation integrated thermal-thermal power system.

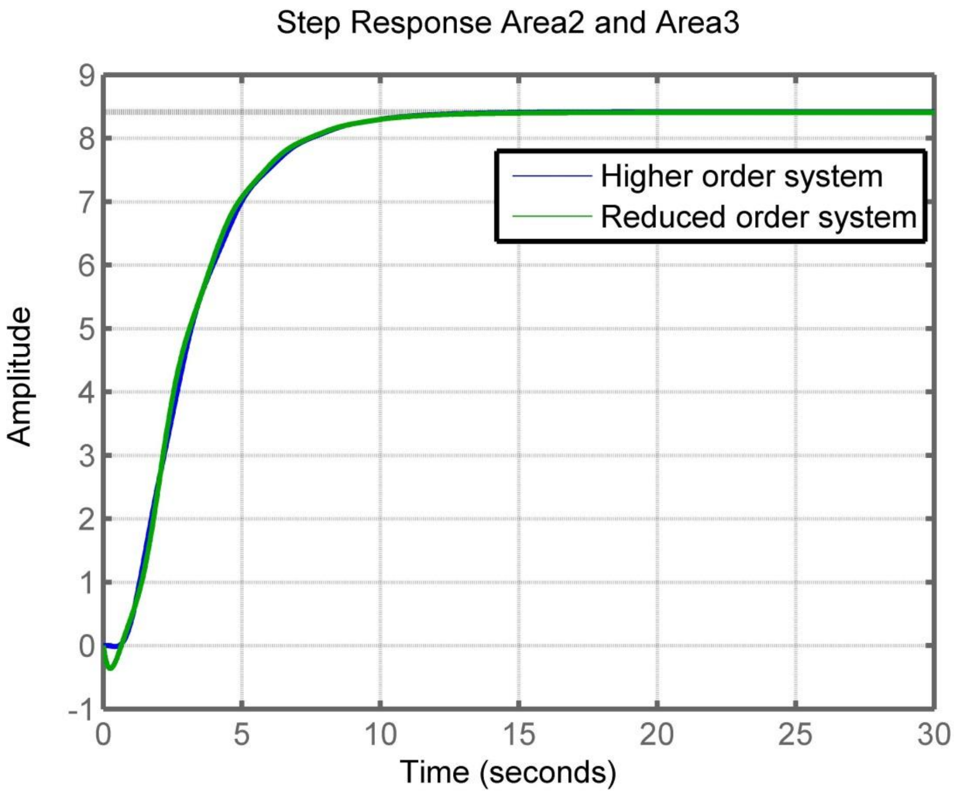


Figure 5.61. Response of the system of high order and low order of area 2 & 3 of the power system of PV-thermal-thermal type containing three control areas for the unit step input

3. For obtaining the 2-dof IMC controller, several operations are performed on the transfer function of the system of reduced-order as follows:

a) To procure controller for $G_1(s)$:

The IMC-PID compensator for the transfer function $G_1(s)$ is obtained by following the proposed methodology and expressed by the following steps:

(i) *Factorization:* Initially, the reduced-order system achieved from the reduction of $G_1(s)$, as shown by equation (5.86a), is factorized in minimal 'G_{1r}^{-}' and non-minimal 'G_{1r}^{+}' parts:

$$G_{1r}(s) = G_{1r}^{-}(s) \cdot G_{1r}^{+}(s) = \left(\frac{2.75s^3 + 0.00019s^2 + 49.6s + 65.3}{s^4 + 3.165s^3 + 13.43s^2 + 21.42s + 7.733} \right) \cdot 1 \qquad (5.87)$$

(ii) *Inversion:* By the inversion of minimal part of the equation (5.87), the initial equation of compensator, i.e. 'Q_{c1}', is attained as depicted in equation (5.88) as follows:

$$Q_{c1}(s) = \left(\frac{2.75s^3 + 0.00019s^2 + 49.6s + 65.3}{s^4 + 3.165s^3 + 13.43s^2 + 21.42s + 7.733} \right)^{-1} \cdot 1 \qquad (5.88)$$

(iii) *Filtration:* A low pass filter is then attached in series with the compensator obtained in equation (5.88) to make the response of the compensator more clear and precise. The low pass filter is here attached to make the compensator strictly proper, and hence, the degree of filtration is two. The overall equation of compensator 'Q_{c1}' from the process is written as follows:

$$Q_{c1}(s) = \left(\frac{2.75s^3 + 0.00019s^2 + 49.6s + 65.3}{s^4 + 3.165s^3 + 13.43s^2 + 21.42s + 7.733} \right)^{-1} \cdot 1 \cdot \frac{1}{(\lambda_1 s + 1)^2} \qquad (5.89)$$

Equation (5.89) has an unknown filter time constant λ_1, which is evaluated by optimizing the behaviour of the system of reduced order with a compensator for the desired output. It is done through the usage of genetic algorithms. After optimisation, the value of λ_1 is obtained as $\lambda_1 = 71.2$, and hence the equation for compensator becomes:

$$Q_{c1}(s) = \frac{s^4 + 3.165s^3 + 13.43s^2 + 21.42s + 7.733}{1.394 \times 10^4 s^5 + 392.6s^4 + 2.514 \times 10^5 s^3 + 3.381 \times 10^5 s^2 + 9348s + 65.3} \qquad (5.90)$$

As it is clear from figure 5.57, area 1 doesn't have any pole that affects load disturbance. So, the 2-dof compensator is not required for area 1, i.e. $G_1(s)$. Hence, it can be described that the compensator obtained from equation (5.90) is sufficient for obtaining the IMC controller that can be used to compensate for the LFC problem occurring in area 1 of the power system shown in figure 5.57.

The IMC controller '$C_1(s)$' for area 1 is then derived from the IMC compensator of equation (5.90) as:

$$C_1(s)=\frac{s^4+3.165s^3+13.43s^2+21.42s+7.733}{1.394\times10^4 s^5+392.6s^4+2.514\times10^5 s^3+3.381\times10^5 s^2+9299s} \quad (5.91)$$

b) *To procure controller for $G_2(s)$ & $G_3(s)$:*

The IMC-PID compensator for the transfer function $G_2(s)$ and $G_3(s)$ is the same and is obtained by following the proposed methodology and expressed by following steps:

(i) *Factorization:* The factorization of the system of reduced-order written by equation (5.86b) is performed and is depicted in equation (5.92):

$$G_{2r}(s)=G_{2r}^{-}(s)\cdot G_{2r}^{+}(s)=\left(\frac{-3.3s^3+8.08s^2-32.8s+65}{s^4+3.165s^3+13.43s^2+21.42s+7.733}\right)\cdot 1 \quad (5.92)$$

(ii) *Inversion:* The next step to obtain the compensator *'Q_{c2}'* is to invert the minimal part *'G_{2r}^{-}'* or invertible part, which yields the equation for IMC compensator written as:

$$Q_{c2}(s)=\left(\frac{-3.3s^3+8.08s^2-32.8s+65}{s^4+3.165s^3+13.43s^2+21.42s+7.733}\right)^{-1}\cdot 1 \quad (5.93)$$

(iii) *Filtration:* A low pass filter is then attached in series of the compensator obtained from equation (5.93) to make it proper. The low pass filter added here is of degree one. The equation (5.94) presents the equation for the IMC compensator after filtration *'Q_{c2}'* as follows:

$$Q_{c2}(s)=\left(\frac{-3.3s^3+8.08s^2-32.8s+65}{s^4+3.165s^3+13.43s^2+21.42s+7.733}\right)^{-1}\cdot 1\cdot\frac{1}{(\lambda_2 s+1)} \quad (5.94)$$

The unknown value of the time constant of the IMC compensator in equation (5.94) *'λ_2'* is then evaluated from the genetic algorithm to optimise the integral square error among the desired output and the obtained output from the compensated system. The value of *'λ_2'* is obtained as $\lambda_2=-709$, and this value provides the equation (5.95) for compensator as follows:

$$Q_{c2}(s)=\frac{s^4+3.165s^3+13.43s^2+21.42s+7.733}{-2.34s^4+2.429s^3-15.8s^2+13.29s+65} \quad (5.95)$$

For designing a 2-dof IMC compensator for the reduction of the effect of disturbance occurring due to load variation, the poles having the effect of load variation are truncated. The truncation is achieved by designing another compensator 'Q_{c3}' having the transfer function as:

$$Q_{c3} = \left(\frac{1 + K_1 s + K_2 s^2 + K_3 s^3 + \cdots\cdots + K_m s^m}{(\lambda_3 s + 1)^m} \right) \tag{5.96}$$

Equation (5.96) has the unknown parameters like degree 'm', gain coefficients of compensator 'K', time constant of disturbance rejecting compensator 'λ_3'. Here, 'm' is the number of poles that are truncated or have the effect of load variation directly on the input. So from figure 5.57, it is clear that in area 2 (Thermal power system), only one pole $s=-20$ has the effect of load variation, and hence the disturbance rejecting compensator 'G_{c3}' becomes:

$$Q_{c3} = \left(\frac{1 + K_1 s}{(\lambda_3 s + 1)^1} \right) \tag{5.97}$$

The motive is to obtain the unknown compensator coefficients 'K_1', 'λ_3' of equation (5.97), which are obtained by optimizing the equation (5.98) by genetic algorithm.

$$\left|1 - G_{2r} \cdot Q_{c2} \cdot Q_{c3}\right|_{s=-1/20} = 0 \tag{5.98}$$

Hence, after optimisation, the value of coefficients are obtained as $K_1 = 20.1164$ *and* $\lambda_3 = 20$. By substituting the values of coefficients, the equation for disturbance rejecting compensator of area 2 power system becomes:

$$Q_{c3} = \left(\frac{1 + 20.1164 s}{1 + 20 s} \right) \tag{5.99}$$

The equation (5.95) and equation (5.99) are then combined to form the 2-DOF compensator for the LFC problem of the 2nd and 3rd area of the given power system, having three control areas. The 2-DOF IMC controller '$C_2(s)$' is then obtained by the combination of equation (5.95) and equation (5.99) as depicted in equation (5.100) as follows:

$$C_2(s) = \frac{20.12s^5 + 64.67s^4 + 273.3s^3 + 444.3s^2 + 177s + 7.733}{-46.8s^5 + 112.2s^4 - 471.9s^3 + 897.9s^2 + 46.09s} \tag{5.100}$$

The equations of controllers achieved in equation (5.91) for area 1 and equation (5.100) for area 2 and area 3 of the power system shown in figure 5.57 are further utilised to form the PID controller and gain parameters of the PID controllers are realized.

4. The IMC controllers obtained in the previous step for all the three areas are then resolved to get the value of gain parameters K_p, K_i, K_d of PID controllers as follows:

a) To procure gain parameters of the 2-dof controller of IMC-PID type for $G_1(s)$:

The ideal PID controller is compared with the 2-DOF controller of area 1 shown in equation (5.91), and gain parameters are recorded. It can be easily understood by the equation (5.101) described as:

$$\frac{s^4 + 3.165s^3 + 13.43s^2 + 21.42s + 7.733}{1.394\times10^4 s^5 + 392.6s^4 + 2.514\times10^5 s^3 + 3.381\times10^5 s^2 + 9299s} = K_p + \frac{K_i}{s} + K_d \cdot s \tag{5.101}$$

The value of the gain parameters of the controller of the IMC-PID type of area 1 is procured from equation (5.101) and tuning the procured controller of PID type, in equation (5.102) as:

$$K_p = -0.1742, K_i = -0.2098, K_d = 0.0962 \tag{5.102}$$

b) To procure gain parameters of the 2-dof-IMC-PID IMC-PID controller for $G_2(s)$ & $G_3(s)$:

Similar to the area 1 controller, the IMC-PID controller for area 2 and area 3 is obtained by comparing the controller designed in equation (5.100) with the ideal PID controller equation as described in equation (5.101), and hence the gain parameters are obtained as recorded in equation (5.103) as follows:

$$\frac{20.12s^5 + 64.67s^4 + 273.3s^3 + 444.3s^2 + 177s + 7.733}{-46.8s^5 + 112.2s^4 - 471.9s^3 + 897.9s^2 + 46.09s} = K_p + \frac{K_i}{s} + K_d \cdot s \tag{5.103}$$

$$K_p = -0.5231\times10^{-06}, K_i = -0.6292\times10^{-06}, K_d = -0.1766\times10^{-06} \tag{5.104}$$

Based on gain parameters obtained in equation (5.102) and equation (5.104), the 2-DOF IMC-PID controllers are added in the feedback of each area (equation (5.102) for area 1 and equation (5.104) for area 2 and area 3). The effect of the load variation on the power system is then checked, and a graph is plotted for the change in frequency and power w.r.t time for the change in load. The results are then compared with the response obtained by employing a PI controller and a controller of the FGS type (Revathi & Mohan Kumar, 2020).

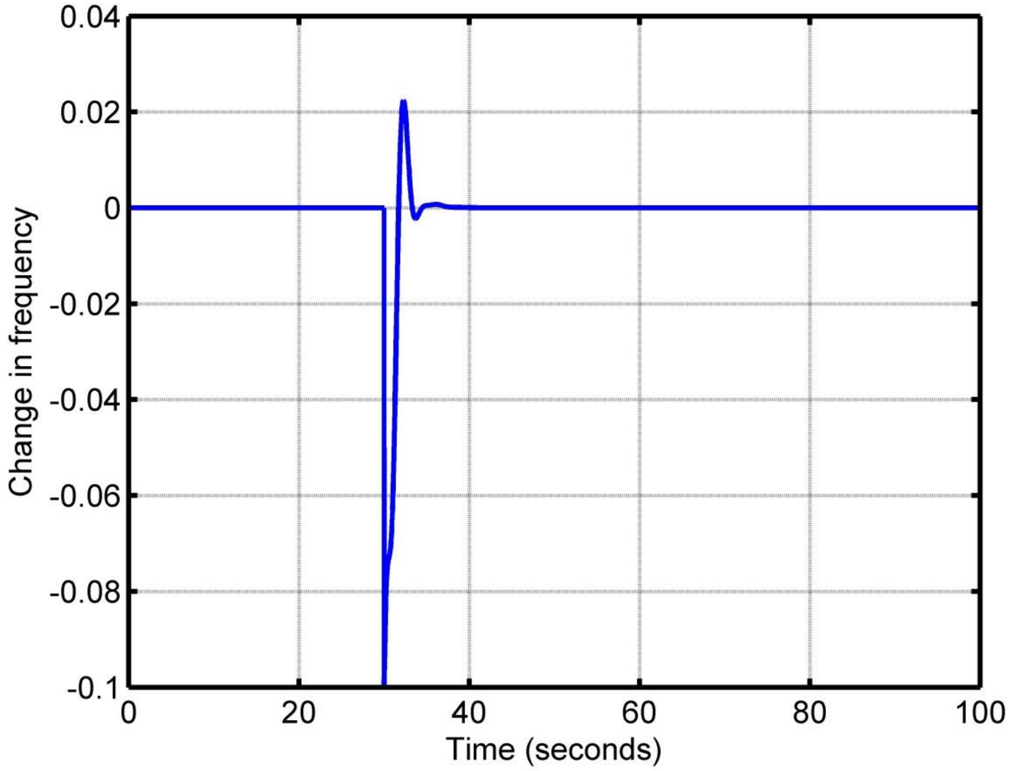


Figure 5.62. Change in frequency with variation in load for area 1 of the power system of PV-thermal-thermal type having three control areas

The implemented PID controller affects the behavioural change in the frequency of area 1, as is observable in figure 5.62 for 10% change in load at 30 seconds. The figure illustrates that after 7.7 seconds, the change in frequency is reduced to zero while the uncompensated system has a frequency deviation equal to 0.1487, as depicted in figure 5.58. Hence, it is revealed that the PID controller eliminates the LFC problem in area 1. While comparing the results obtained by Revathi, the PI controller stabilizes the frequency deviation after 66 seconds, and the fuzzy gain scheduling controller achieves the zero frequency deviation after 61.5 seconds. Also, while comparing the undershoot occurring at the time of load variation, it is -0.1 for the implemented PID controller, -0.1781 for PI controller and -0.1404 for fuzzy gain scheduling (FGS) controller. So by employing implemented controller, undershoot is also decreased, providing more exciting results. Hence, it is clear from the analysis that the implemented PID controller eliminates the effect of load variation on the frequency of operation of area 1 with a better time of settlement and overshoot as compared to the PI controller and FGS controller.

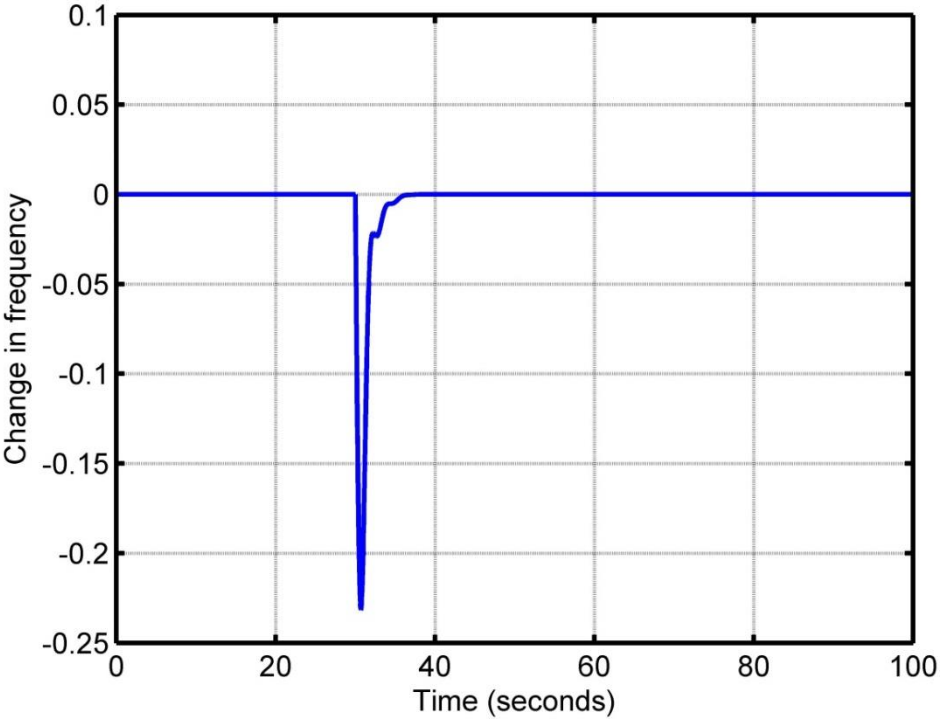


Figure 5.63. Change in frequency with variation in load for area 2 of the power system of PV-thermal-thermal having three areas

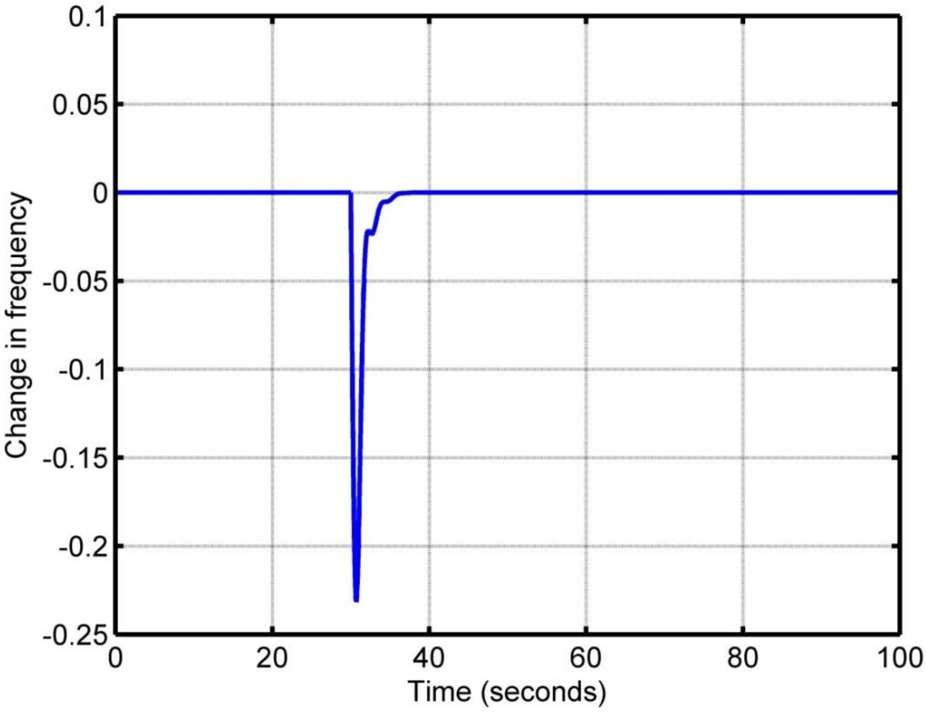


Figure 5.64. Change in frequency with variation in load for area 3 of the power system of PV-thermal-thermal type having three areas

Figure 5.63 clears that for 10% change in load on area 2 at 30 seconds, the frequency deviation is reduced to zero after 7.78 seconds compared to 75 seconds for PI controller and 62 seconds in FGS controller as shown by Revathi. But the undershoot exhibited by implemented PID controller is -0.2316, which is a little bit higher than -0.2077 for the FGS controller and -0.2128 for the PI controller. The increased undershoot can be compensated by the highly decreased value of settling time. It reveals that the controller implemented performs better than the controller of PI type and controller of FGS type designed by Revathi.

Similarly, by comparing the frequency deviation curve of area 3 in figure 5.64, the settling time is observed as 7.78 seconds, and undershoot is -0.2316. From the analysis of results shown by Revathi, it is clear that the settling time for PI controller is 68 seconds and for FGS controller is 62 seconds, and the value of undershoot for PI controller is -0.19 and for FGS controller is -0.1572. So it can be revealed for area 3 that undershoot in the implemented PID controller is slightly higher. Still, the settling time is reduced to a great extent and hence implemented PID controller can be described better than the PI controller and FGS controller.

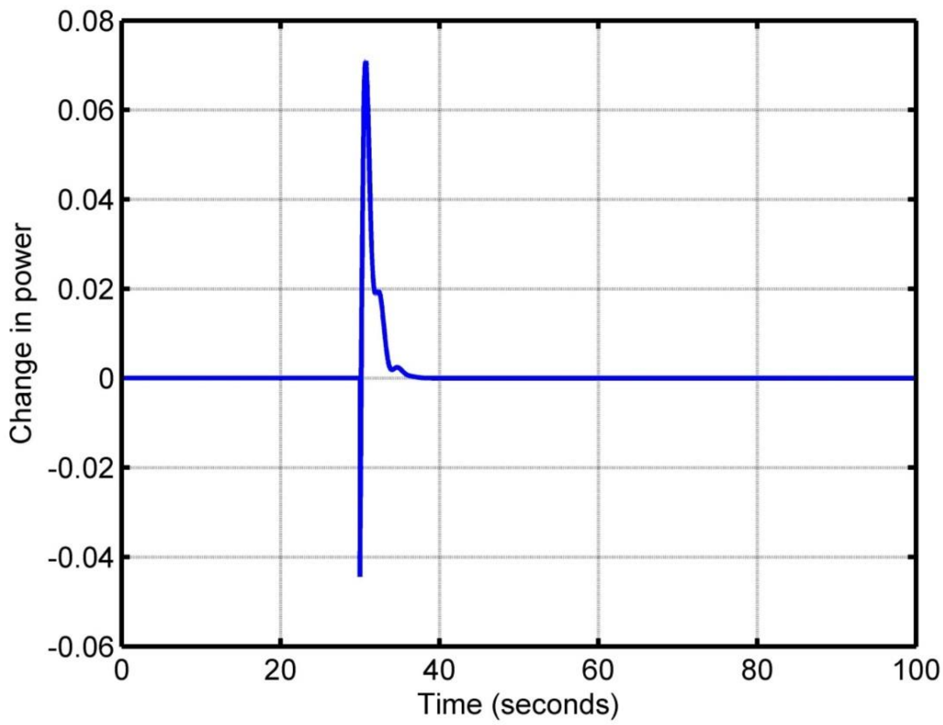


Figure 5.65. Change in power with variation in load at tie-line of the power system of PV-thermal-thermal type having three areas

Figure 5.65 depicts the change in tie-line power due to 10% load variation at 30 seconds, which describes that after almost 7.3 seconds of load variation, the effect on power eliminates, providing the first overshoot of value 0.042. These values are better than the PI controller (with settling time 64 s, the first shoot is 0.058) and the FGS controller (with settling time 58 s, the first shoot is 0.042).

While comparing the existing methods of LFC with the implemented method, it is analysed that (Azeer et al., 2017) designed three types of controllers to remove the LFC problem in 1 two area power system. The controllers were designed by PID, fuzzy and ANN approaches. These controllers provided a considerable settling time of 40-50 seconds for different controllers in different areas. The proposed IMC-PID controller performs the LFC in a short period of 10 seconds. Also, the overshoot in the Azeer controller is very high as compared to the proposed controller. By using the concept of model predictive control (MPC), LFC controllers have been designed by (Kunya et al., 2019; Liu et al., 2019), which again provided a considerable settling time of around 30 seconds and overshoot in frequency is about 0.05 and fewer oscillations. The implemented controller performs better than the controller designed in literature providing low settling time, less shoot value as depicted by figure (5.62-5.65). (Shakibjoo, Moradzadeh, & Moussavi, 2020) added a proportional-derivative (PD) controller to the fuzzy controller using multilayer perceptron ANN. The settling time by the designed controller was reduced to almost 3-4 seconds but the value of overshoot is increased to 0.06, which is very much higher as compared to the implemented controller. The incorporation of a sliding mode controller for the frequency regulation was done by (A. Kumar et al., 2021). The controller designed here takes a settling time equal to 3 seconds to 60 seconds to stabilise the frequency depending on the type of power system. But the shortcoming with the design is that the controller is designed for the power system having a single area and two areas.

In comparison, the implemented controller is designed for a power system consisted of three areas. The parameters of a PID controller are tuned by (D. K. Gupta et al., 2021) by using a hybrid optimisation algorithm which is a combination of three algorithms such as gravitational search algorithm, particle swarm optimisation and firefly algorithm. This hybrid controller possesses a little lower value of settling time equal to 8.5 seconds than 10 seconds for the proposed controller. The undershoot value is also lower than the proposed controller. But the controller presented by Gupta is implemented on a power system having two areas.