CHAPTER 4
METHODOLOGY

For obtaining the objectives, a novel blended method named the IPCG method for reducing the order of a system is developed in the research. The IPCG technique is also utilised to procure a 2-DOF controller for LFC problem elimination. The methodology drawn to perform the two operations is described as follows:

4.1 METHODOLOGY FOR REDUCING MODEL ORDER

The IPCG method is used to reduce the order of a system that is a blended type method of reducing the order of a model and is formed by combining two techniques: (i) improved pole clustering (ii) genetic algorithm. The technique of improved pole clustering is utilised to procure the denominator multinomial of the system of reduced order. The denominator multinomial fetched from enhanced pole clustering then undergoes a process of optimisation through a genetic algorithm, and then the numerator of the system of reduced-order is procured. The complete progression to fetch the reduced-order approximation by the IPCG technique is presented by different operation steps. In the clustering technique, the poles of the system of high order are combined to form the clusters. Then using different approaches, centres of these clusters are procured, and these cluster centres act as the poles of the system of reduced order. There are numerous disadvantages associated with the existing clustering approaches that the wrong selection of cluster centre will result in improper approximation. Moreover, the effect of some poles may also be neglected by improper clustering. These limitations are taken into consideration in the IPCG method such that all poles of the system of high order are consumed in forming the cluster centre. Moreover, the clusters are formed based on their model dominance index (MDI). According to the IPCG method of reduction, the poles of the system of high order are arranged in the decreasing order of their dominancy. Then clusters are constructed by placing the poles in the cluster based on the dominancy index as described in the detailed description of the methodology. The cluster centres of these clusters are then procured by Lehmer's measure criterion (Aguirre, 1993) by simple mathematical calculation. The cluster centres hence formed serve as the poles of the system of reduced order. The steps to obtain the model order reduction are presented as follows:

a) Obtain the poles of the system of higher-order

Consider a system of higher-order that is presented in the form of transfer function representation as shown through equation (4.1). The numerator coefficients are represented by $x = \{x_0, x_1, x_2, ..., x_{a-1}, x_a\}$, and the denominator coefficients are represented as $y = \{y_0, y_1, y_2, ..., y_{b-1}, y_b\}$. Assuming that the order of the system of higher-order is *'b'* and hence the poles in the system are *'b'*, and total zeros in the given system are *'a'*. The first step to perform the enhanced pole clustering is to procure the poles of the system of higher-order. To procure these poles, the denominator of the system is to be factorized as obtained in equation (4.2). Hence, the poles in the system of higher-order are obtained as $\sigma_1, \sigma_2, \sigma_3, \ldots\ldots, \sigma_{b-1}$ and σ_b.

$$G_l(s) = \frac{x_0 s^a + x_1 s^{a-1} + x_2 s^{a-2} + ...+ x_{a-1}s + x_a}{y_0 s^b + y_1 s^{b-1} + y_2 s^{b-2} + ...+ y_{b-1}s + y_b} \tag{4.1}$$

$$G_l(s) = \frac{x_0 s^a + x_1 s^{a-1} + x_2 s^{a-2} + ...+ x_{a-1}s + x_a}{(s-\sigma_1)\cdot(s-\sigma_2)\cdot(s-\sigma_3)\cdot......\cdot(s-\sigma_{b-1})\cdot(s-\sigma_b)} \tag{4.2}$$

b) Procure the residues of the system of higher-order

The next step for fetching the reduction in the order of a model is to get the residues of the system shown by equation (4.1). It is achieved by expanding the equation (4.2) by a partial fraction. The expansion through a partial fraction of equation (4.2) is depicted by equation (4.3).

$$G_l(s) = \frac{\rho_1}{(s-\sigma_1)} + \frac{\rho_2}{(s-\sigma_2)} + \frac{\rho_3}{(s-\sigma_3)} + ...+ \frac{\rho_{b-1}}{(s-\sigma_{b-1})} + \frac{\rho_b}{(s-\sigma_b)} \tag{4.3}$$

From the method of partial fraction, the *'b'* residues of the system are obtained as $\rho_1, \rho_2, \rho_3, \ldots, \rho_{b-1}, \rho_b$. It is possible that the higher-order system may be formed of only real poles or only complex poles or maybe consisted of the blended system of real and complex poles. So, the residues procured in the equation (4.3) may also be real or complex or both depending on the type of pole.

c) Obtain the model dominancy index of the system of high order

The next step of the task is to obtain the value of the model dominance index (MDI) of each pole present in the system of equation (4.1). The value of MDI is obtained according to the type of pole, which is described as follows:

(i) When all poles of the system are real:

When all poles of the equation (4.2) are real, then the residues of the equation (4.3) are also considered real, i.e. σ, ρ are real-valued. Hence the MDI for the i^{th} pole is written as:

$$\eta_i = -\frac{\rho_i}{\sigma_i} \text{ s.t. } i=1,2,3,.....,b-1,b \qquad (4.4)$$

Hence all *'b'* MDIs are obtained by equation (4.4), and these MDIs are to be used further in the next step.

(ii) When all poles of the system are complex:

When all poles of the equation (4.2) are complex, then the residues of the equation (4.3) are also considered complex, i.e. σ, ρ are complex-valued. As the complex poles exist in the complex conjugate pairs, so the equation (4.3) can be represented in the form of equation (4.5) having only complex poles as:

$$G_l(s) = \frac{\rho_1}{(s-\sigma_1)} + \frac{\rho_1 *}{(s-\sigma_1 *)} + \frac{\rho_2}{(s-\sigma_2)} + \frac{\rho_2 *}{(s-\sigma_2 *)} ....+ \frac{\rho_b}{(s-\sigma_b)} + \frac{\rho_b *}{(s-\sigma_b *)} \qquad (4.5)$$

The equation (4.5) is further rearranged by combining the complex conjugate pairs as:

$$G_l(s) = \frac{(\rho_1 + \rho_1 *)\cdot s - (\rho_1 \cdot \sigma_1 * + \rho_1 * \cdot \sigma_1)}{(s-\sigma_1)\cdot(s-\sigma_1 *)} + \frac{(\rho_2 + \rho_2 *)\cdot s - (\rho_2 \cdot \sigma_2 * + \rho_2 * \cdot \sigma_2)}{(s-\sigma_2)\cdot(s-\sigma_2 *)} + ...$$
$$+ \frac{(\rho_{b/2} + \rho_{b/2} *)\cdot s - (\rho_{b/2} \cdot \sigma_{b/2} * + \rho_{b/2} * \cdot \sigma_{b/2})}{(s-\sigma_{b/2})\cdot(s-\sigma_{b/2} *)}$$

Hence the MDI for the i^{th} complex pole is written as:

$$\eta_i = -\frac{\rho_i \sigma_i * + \rho_i * \sigma_i}{\sigma_i \sigma_i *} \text{ s.t. } i=1,2,3,.....,b/2-1,b/2 \qquad (4.6)$$

Hence all *'b'* MDIs are obtained by equation (4.6), and these MDIs are to be used further in the next step.

(iii) When poles are real and complex, both:

The original system of equation (4.3) has *'m'* real poles and *'n'* complex poles existing in complex conjugate form. Then the original system can be written after rearrangements in equation (4.7) as follows:

$$
\begin{aligned}
G_l(s) = {} & \frac{\rho_1}{(s-\sigma_1)} + \frac{\rho_2}{(s-\sigma_2)} + \dots\dots\dots\dots + \frac{\rho_m}{(s-\sigma_m)} + \frac{(\rho_{m+1} + \rho_{m+1}*)\cdot s - (\rho_{m+1}\cdot\sigma_{m+11}* + \rho_{m+1}*\cdot\sigma_{m+1})}{(s-\sigma_{m+1})\cdot(s-\sigma_{m+1}*)} \\
& + \frac{(\rho_{m+2} + \rho_{m+2}*)\cdot s - (\rho_{m+2}\cdot\sigma_{m+2}* + \rho_{m+2}*\cdot\sigma_{m+2})}{(s-\sigma_{m+2})\cdot(s-\sigma_{m+2}*)} + \dots\dots\dots\dots \\
& + \frac{(\rho_{m+n/2} + \rho_{m+n/2}*)\cdot s - (\rho_{m+n/2}\cdot\sigma_{m+n/2}* + \rho_{m+n/2}*\cdot\sigma_{m+n/2})}{(s-\sigma_{m+n/2})\cdot(s-\sigma_{m+n/2}*)}
\end{aligned}
\tag{4.7}
$$

So, there will be *'m'* MDIs for the poles of real value and *'n'* MDIs for the poles of the complex value. Then, MDI for each actual pole is obtained by equation (4.4), and the value of MDI for complex poles are obtained by equation (4.6).

d) Ordering

The next step in obtaining the reduced-order system is to arrange the MDIs in a particular order. The value of MDI obtained from equation (4.4) and equation (4.6) for the poles of real and complex values, respectively, are arranged in the plummeting order. Higher dominancy is placed first in the row, and the lowest dominancy value is placed last in the row. From equation (4.8), it is easily understood that the first MDI 'η_1' is considered to have the highest value, and the last MDI 'η_b' has the lowest value.

$$
|\eta_1| \succ |\eta_2| \succ |\eta_3| \succ |\eta_4| \succ \dots\dots\dots\dots \succ |\eta_{b-1}| \succ |\eta_b| \tag{4.8}
$$

The ordering of MDIs is to be done carefully so that the clusters can be made properly.

e) Clustering

The number of clusters to be made equates to the order of the system of reduced order. Each cluster is consists of one or more poles of the system of a high order. The cluster is formed by placing the poles in the cluster based on the MDI order obtained in equation (4.8). The pole with the highest dominancy is placed in the first cluster and consists of some consecutive poles and the first cluster. The second cluster starts with the pole placed next to the last pole of the first cluster. The procedure continues to form the *‘r’* clusters. It is to be noted while making clusters that the real and complex poles will be placed in separate clusters, and no cluster contains repeated poles. Also, it is to be considered that one pole can’t be placed in two different clusters.

f) *Formation of denominator multinomial*

After forming the cluster, a cluster centre is obtained for each cluster, which acts as a pole of the reduced denominator polynomial. Hence, the cluster centre is procured by Lehmer’s measure criterion, which measures the dominancy of poles and neglected the dependency on the time moments of the system. The significant advantage of using Lehmer’s measure criterion is that the fast and slow modes are considered to form the cluster centre. According to poles, there are three types of systems; these are only real poles systems, only complex poles systems, the blended system having real and complex poles. So the equation of denominator multinomial for the three systems is obtained as follows:

(i) *If the higher-order system has only real poles:*

Considering that the higher-order system has only real poles, all clusters' cluster centres will only be real. Suppose the cluster formed in the previous step contains ‘z’ poles arranged in the equation (4.8). Also, the highest prominent pole in the cluster is *‘σ_1’*. Then the centre of the cluster, i.e. *‘σ_c’* of the real pole cluster, is obtained by solving equation (4.9) as described follows:

$$\sigma_c = \left[\frac{\left(\frac{1}{\sigma_1}\right) + \sum_{p=2}^{z}\left(\frac{1}{\sigma_p}\right)}{\left(\frac{1}{\sigma_1}\right)^2 + \sum_{p=2}^{z}\left(\frac{1}{\sigma_p}\right)^z} \right]^{z-1} \qquad (4.9)$$

For all the *'r'* clusters of the r^{th} order reduced system, the cluster centre of all *'r'* clusters is obtained by equation (4.9). Hence, the *'r'* cluster centres are combined to form a reduced-order denominator multinomial as follows:

$$\begin{aligned} D_r(s) &= (s-\sigma_{c1})\cdot(s-\sigma_{c2})\cdots\cdot(s-\sigma_{cr-1})\cdot(s-\sigma_{cr}) \\ &= s^r + n_{r-1}s^{r-1} + n_{r-2}s^{r-2} + \cdots\cdots + n_1 s + n_0 \end{aligned} \tag{4.10}$$

(ii) *If the higher-order system has only complex poles:*

Considering that the higher-order system has only complex poles *(ς+iψ),* the reduced-order denominator multinomial consists of only complex poles and the cluster centre of all clusters is also complex. The cluster centre of the cluster is obtained by separately determining the real and imaginary parts of the cluster centre. The real part of the cluster centre is obtained by combining the real parts of the poles existing in the cluster. In contrast, the imaginary portion of the cluster centre is procured by the imaginary part of poles in the cluster by solving equations (4.11) and (4.12), respectively.

$$\zeta_c = \left[\frac{\left(\frac{1}{\zeta_1}\right) + \sum_{p=2}^{z}\left(\frac{1}{\zeta_p}\right)^{z-1}}{\left(\frac{1}{\zeta_1}\right)^2 + \sum_{p=2}^{z}\left(\frac{1}{\zeta_p}\right)^{z}} \right] \tag{4.11}$$

$$\psi_c = \left[\frac{\left(\frac{1}{\psi_1}\right) + \sum_{p=2}^{z}\left(\frac{1}{\psi_p}\right)^{z-1}}{\left(\frac{1}{\psi_1}\right)^2 + \sum_{p=2}^{z}\left(\frac{1}{\psi_p}\right)^{z}} \right] \tag{4.12}$$

All cluster centres are then combined to form the reduced-order denominator multinomial as follows:

$$\begin{aligned} D_r(s) = &(s-\zeta_1-i\psi_1)\cdot(s-\zeta_1+i\psi_1)\cdot(s-\zeta_2-i\psi_2)\cdot(s-\zeta_2+i\psi_2)\cdots\cdots \\ &(s-\zeta_{r/2}-i\psi_{r/2})\cdot(s-\zeta_{r/2}+i\psi_{r/2}) \end{aligned} \tag{4.13}$$

(iii) *If the higher-order system has both real and complex poles:*

In this case, the cluster centre of the real poles cluster is obtained by equation (4.9), and the cluster centre of the complex poles cluster is obtained by equations

(4.11) and (4.12). Then real cluster centres and complex cluster centres are combined to obtain the equation of denominator multinomial as follows:

$$D_r(s) = (s-\sigma_1)\cdot(s-\sigma_1)\cdots(s-\sigma_a)\cdot(s-\zeta_{a+1}-i\psi_{a+1})\cdot(s-\zeta_{a+1}+i\psi_{a+1})\cdot (s-\zeta_{a+2}-i\psi_{a+2})\cdot(s-\zeta_{a+2}+i\psi_{a+2})\cdots\cdots(s-\zeta_{r/2}-i\psi_{r/2})\cdot(s-\zeta_{r/2}+i\psi_{r/2}) \tag{4.14}$$

The equation for a denominator is obtained in equations (4.10), (4.13) & (4.14), which is employed to procure the numerator multinomial of the system of the reduced order.

g) Formation of numerator multinomial

The equation of the reduced-order system is obtained after obtaining the value of denominator multinomial, as shown in equation (4.15). The coefficients of the numerator, i.e. $(m_{r-1}, m_{r-2}, \cdots\cdots, m_1, m_0)$ are unknown.

$$G_r(s) = \frac{m_{r-1}s^{r-1} + m_{r-2}s^{r-2} + \cdots\cdots + m_1 s + m_0}{s^r + n_{r-1}s^{r-1} + n_{r-2}s^{r-2} + \cdots\cdots + n_1 s + n_0} \tag{4.15}$$

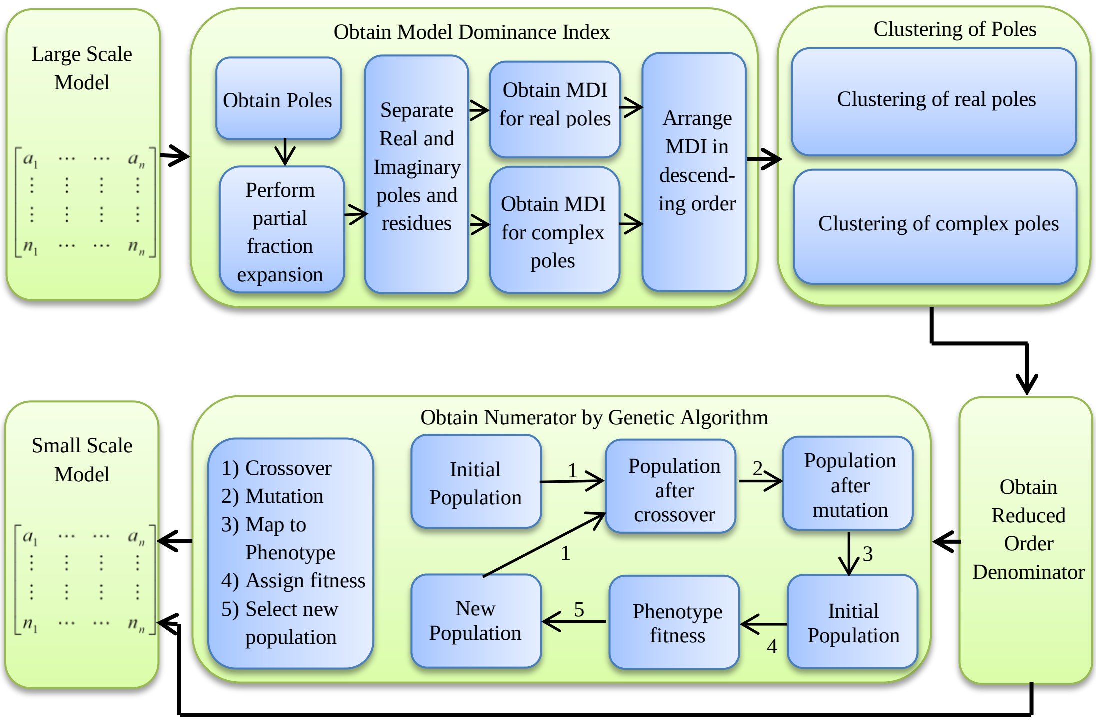

Figure 4.1. Flowchart of the methodology developed in the research work

The value of these coefficients is obtained by optimisation of equation (4.15) by genetic algorithm. The fitness function in the optimisation is ISE, which represents error amongst the response of the system of higher-order and of the approximated system of reduced-order when applied input is unit step as shown in equation (4.15). Genetic algorithm is a process of random global searching designed to select naturally and then to do evolution. An initial population is employed to sample the complete space and then a selection process is possessed to evaluate the population depending on the fitness standard of each individual of the members of population known as chromosomes. A number of generations are utilized for convergence and then the new population is formed at each generation based on the fitness of prior generation. There are general four aspects of genetic algorithm namely fitness/ selection, mutation, crossover and objective function. Initial population is the foremost point of the process of GA, where a set of binary strings, referred to as population or genotype or chromosomes and hence each single element forms the optimization problem's potential solution. The alteration in chromosomes is performed according to the optimisation function. The major role of GA depends on the crossover operator for the exploitation and mutation operator for exploration. The crossover operator collects the part of two good parent chromosomes aiming to produce the younger global minimum. The mutation operator employs inversion of binary bits (for binary string of population) for completion of task of optimisation. The proposed methodology used real coded genetic algorithm employing a double vector type population. Hence, using genetic optimisation, the minimum value of ISE is obtained and further determining the optimised value of unknown numerator coefficients. The crossover and mutation is constraint dependant in the proposed methodology. Also, a population of double vector type is used here with the population size of 50 in each example discussed by the proposed methodology. The selection criterion is stochastic, and the rank type scaling function is used here. The complete approach to perform the chore of reducing the order of a model is highlighted by a flow chart shown in figure 4.1.

In brief, the complete methodology to fetch the lower order approximation of the system of higher order is a blend of pole clustering and genetic algorithm. Initially the system of higher order is separated in the numerator and denominator multinomial. In the first phase, the denominator multinomial of the system of higher order is reduced by improved pole clustering method based on the value of MDI of each pole. The approximated numerator multinomial of order one less than the order of multinomial of reduced order is formed by approximating the coefficients of numerator. This approximated numerator is then combined with the denominator multinomial of reduced order and the combined equation is optimised by genetic algorithm. The fitness function of genetic algorithm is taken as ISE among the system of higher order and the approximated system of lower order. The optimised value of numerator coefficients is then obtained from the optimisation process, which is fetching the system of reduced order. It is supposed during the process of order reduction that the performance parameters of the system of a high order must be preserved in the equivalent system of reduced order. So, the higher and reduced systems are compared based on the performance parameters in the time domain and frequency domain as written below:

a) ***Integral square error (ISE):*** The slightest error between the higher and reduced system is detected by the ISE because ISE takes the sum of the square of error. A small amount of error gets squared and contributes to comparing the two systems. This error is represented as follows:

$$ISE = \int \left[c(t) - c_r(t)\right]^2 .dt$$

Here, *c(t)* represents the output of the system of high order and $c_r(t)$ represents the output of the approximation of reduced-order procured by the MOR technique.

b) ***Integral time absolute error (ITAE):*** The value of error existing amongst the given system of high order and procured approximation of reduced-order is multiplied with time before taking its sum. This error is preferable to detect the error which takes place after long time intervals. This error is computed as:

$$ITAE = \int t \cdot \left[c(t) - c_r(t)\right] .dt$$

c) ***Integral absolute error (IAE):*** This error is obtained by simply finding the difference between two outputs, i.e. higher-order and reduced-order systems. This error computed the absolute value of the difference in the two systems and represented as:

$$IAE = \int \left|c(t) - c_r(t)\right| .dt$$

d) ***Rise time:*** The time taken by the system's reaction to deviate from the minimum value of the output to the steady-state value.

e) ***Overshoot:*** Overshoot is the presentation of a signal which exceeds its target value. In control systems, overshoot is the value of response when it exceeds its steady-state value.

f) ***Settling time:*** The time taken by the reaction to arrive at the consistent state and stay inside the predetermined resistance groups around the last worth. As a rule, the resistance groups are 2% and 5%.

g) ***Steady-state error:*** The difference between the response of a given system and obtained system in a steady domain is computed as a steady-state error. This error should be 'zero' for better reduced-order approximation.

h) ***Gain margin (GM):*** The measure of progression in the gain of an open-loop system to construct an unstable closed loop system is termed the gain margin. The distinction between unity gain and the value of gain at the frequency of phase crossover provides a period of $-180°$.

i) ***Phase margin (PM):*** The measure of progression in the open-loop system to construct an unstable system of closed-loop is termed as phase margin. The distinction between the -180° phase and the phase at the frequency of gain crossover gives a margin of 0 dB.

So, if the value of error amongst the given system of high order in equation (4.1) and the procured system of reduced-order in equation (4.15) is lowest, then the two systems can be termed equivalent. Also, the similar value of rising time, overshoot, settling time, GM, PM signifies that the two systems are approximately similar. Hence, the reduced-order system can be used instead of a system of high order for the computations and behaviour analysis.

Moreover, mathematically, the equivalency between the two systems is also computed by obtaining their first three-time moments. The given system and the system obtained after model order reduction is said to be similar if their time moments are approximately similar. Considering that a system is represented in general form by its transfer function *G(s)* as:

$$G(s) = \frac{\sum_{i=0}^{m} a_k s^i}{\sum_{j=0}^{n} b_k s^j} \tag{4.16}$$

The coefficients *'a_k'* and 'b_k' describe the numerator and denominator characteristics, respectively. The value of *'a_k'* describes that a system is a minimum phase or non-minimum phase, while the value of 'b_k' shows that the system is stable or not. Now, the i^{th} time moment *'T_i'* of the system shown in equation (4.16) is obtained by solving the equation (4.17) described as:

$$T_i = \left. \frac{dG(s)}{dt} \right|_{s=0} \tag{4.17}$$

So to check the equivalency of the higher and reduced-order model obtained by the proposed method, three initial time moments of both the systems are obtained and compared. If both systems' values of time moments are equal, then these two systems are said to be equivalent or similar.

The same methodology is applied to reduce the MIMO systems as the MIMO system is comprised of several SISO systems. So all SISO systems are fetched from the MIMO system, and each SISO system is reduced individually. Then, the reduced SISO systems are combined back to form the MIMO system of reduced order. Moreover, the discrete systems are reduced by the same methodology. Initially, the discrete system is converted into a continuous system by the transformation principle. Then the continuous system is reduced by the proposed IPCG technique. After that, the reduced continuous system is converted into the corresponding discrete system by inverse transformation, and hence discrete system's reduced approximation is procured.

4.2 METHODOLOGY FOR DESIGN OF LFC CONTROLLER

The proposed design of the LFC controller employs the controller of two degrees of freedom, which employs two separate controllers embedded in a single control system as indicated by the schematic diagram of figure 4.2. The first controller is an internal model control (IMC) based, and the second controller is designed directly to reject the disturbance occurring in the system. The design procedure of both controller and the method to combine them is described as follows:

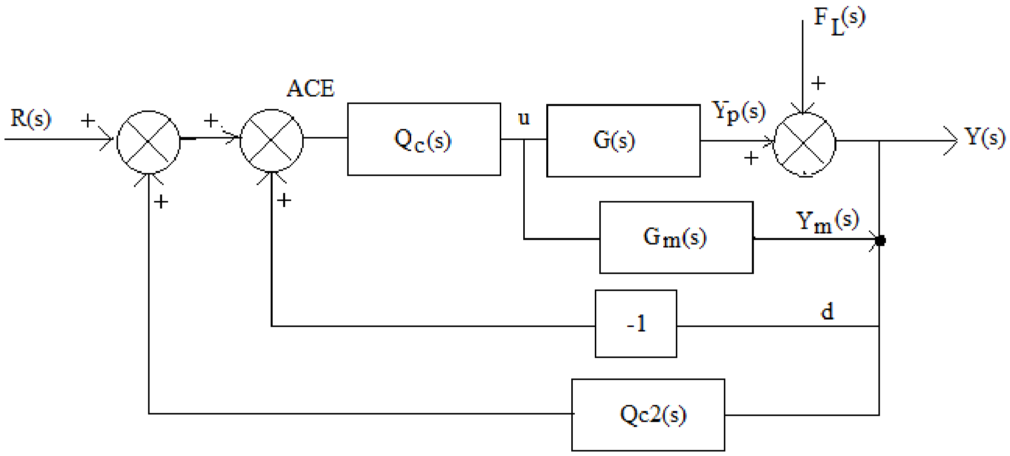

Figure 4.2. Schematic diagram showing the basic structure of the power system containing 2-DOF controller

a) Obtaining the total transfer function of the system

The basic IMC structure of figure 1.9 shows that two inputs contribute to the process model output. These inputs are a) applied input to the system b) disturbance entering into the system. Hence, it becomes necessary to find the combined response of both the inputs. Consider the input-output relation for both cases can be represented in the frequency domain by their transfer function as follows:

$$G_i(s) = \frac{Y_p(s)}{R(s)} = \frac{a_m s^m + a_{m-1}s^{m-1} + a_{m-2}s^{m-2} + \cdots\cdots + a_1 s + a_0}{b_n s^n + b_{n-1}s^{n-1} + b_{n-2}s^{n-2} + \cdots\cdots + b_1 s + b_0} \quad (4.18)$$

$$G_d(s) = \frac{Y_d(s)}{F_L(s)} = \frac{e_k s^k + e_{k-1}s^{k-1} + e_{k-2}s^{k-2} + \cdots\cdots + e_1 s + e_0}{b_n s^n + b_{n-1}s^{n-1} + b_{n-2}s^{n-2} + \cdots\cdots + b_1 s + b_0} \quad (4.19)$$

Equation (4.18) is the power system transfer function w.r.t. applied input only, and equation (4.19) is the power system transfer function w.r.t disturbance input. The output of the dynamic power system is a combination of *'$Y_p(s)$'* and *'$Y_d(s)$'* outputs, which are obtained from inputs $R(s)$ and $F_L(s)$. The inputs' combined output is termed as process output *'Y'*, which is obtained as $Y=Y_p(s) + Y_d(s)$. Alternately, the combined transfer function $G_l(s)$ of the power system, i.e. equation (4.20), is said to be a combination of both the transfer functions and can be written as $G_l(s)=G_i(s)+G_d(s)$.

$$G_l(s) = \frac{Y(s)}{X(s)} = \frac{f_x s^x + f_{x-1} s^{x-1} + f_{x-2} s^{x-2} + \cdots\cdots + f_1 s + f_0}{b_n s^n + b_{n-1} s^{n-1} + b_{n-2} s^{n-2} + \cdots\cdots + b_1 s + b_0} \qquad (4.20)$$

So the transfer function of a higher-order system with load disturbance is obtained by the combination transfer function in equation (4.20), which is reduced by the MOR technique developed.

b) Reducing the order of the transfer function

The system's transfer function presented by equation (4.20) is reduced through the methodology presented earlier in section 4.1. After reduction, a reduced-order approximated system is hence obtained as represented in equation (4.15). Then the performance characteristics of the system given in transfer function form as shown in equation (4.20) and the system presented in the transfer function form as in equation (4.15) are compared. If these characteristics are approximately similar, then the reduced-order system of equation (4.15) can be utilised for the design of the controller instead of a higher-order system of equation (4.20).

c) Design of internal model control based compensator

The IMC controller designing is done based on the following steps:

(i) Factorization: A transfer function is assumed to have two parts in it. One is a non-minimal part, while another is a minimal part. The non-minimal part consists of the poles existing in the right s-plane, while the minimal portion consists of the poles existing in the left s-plane. Initially, the transfer function obtained in equation (4.15) is factorized in two parts: minimal $G_r^-(s)$ and non-minimal $G_r^+(s)$ as shown in equation (4.21):

$$G_r(s) = G_r^-(s) \cdot G_r^+(s) \qquad (4.21)$$

(ii) Inversion: The minimal part is invertible, while the non-minimal part is non-invertible. The minimal part is inverted in this step while keeping the non-minimal

part intact, which forms an IMC compensator. It is clearly understood from the equation (4.22).

$$Q_{c1}(s) = \left[G_r^-(s)\right]^{-1} \cdot G_r^+(s) \tag{4.22}$$

(iii) *Filtration:* The compensator obtained in equation (4.22) is an improper compensator, so a filter of passing lower frequencies is added in the cascade with the compensator to make it semi-proper or proper, as shown in equation (4.23). As shown in equation (4.24), the low pass filter *F(s)* has the unknown value of filter time coefficient 'λ_1' obtained by optimisation of the compensated system by genetic algorithm. And the value of *'x'* is chosen so that the compensator equation becomes semi-proper or proper.

$$Q_{c1}(s) = \left[G_r^-(s)\right]^{-1} \cdot F(s) \tag{4.23}$$

$$F(s) = \frac{1}{(\lambda_1 s + 1)^x} \tag{4.24}$$

The compensator obtained in equation (4.23) is termed as IMC compensator.

d) Design of disturbance rejection compensator

Sometimes, obtaining the value of only one parameter, *'λ_1'*, is not enough to reduce the effect of disturbance caused by load variation. So, one more controller is required to be designed to improve the load disturbance rejection parameter. The second controller is also added to the feedback of the system. The disturbance rejection compensator is represented in the equation (4.25)

$$Q_{c2}(s) = \frac{(1 + k_1 s + k_2 s^2 + \cdots\cdots + k_{m-1} s^{m-1} + k_m s^m)}{(1 + \lambda_2 s)^m} \tag{4.25}$$

The parameter *'λ_2'* is the tuning parameter of the compensator that removes the disturbance. The coefficients and parameters in equation (4.25) are chosen to cancel the poles having load disturbance. Suppose there are *'m'* numbers of a pole that are to be rejected ($p_1, p_2, \ldots, p_{m-1}, p_m$) then the coefficients $k_1, k_2, \ldots, k_{m-1}, k_m$ are obtained by satisfying the equation (4.26).

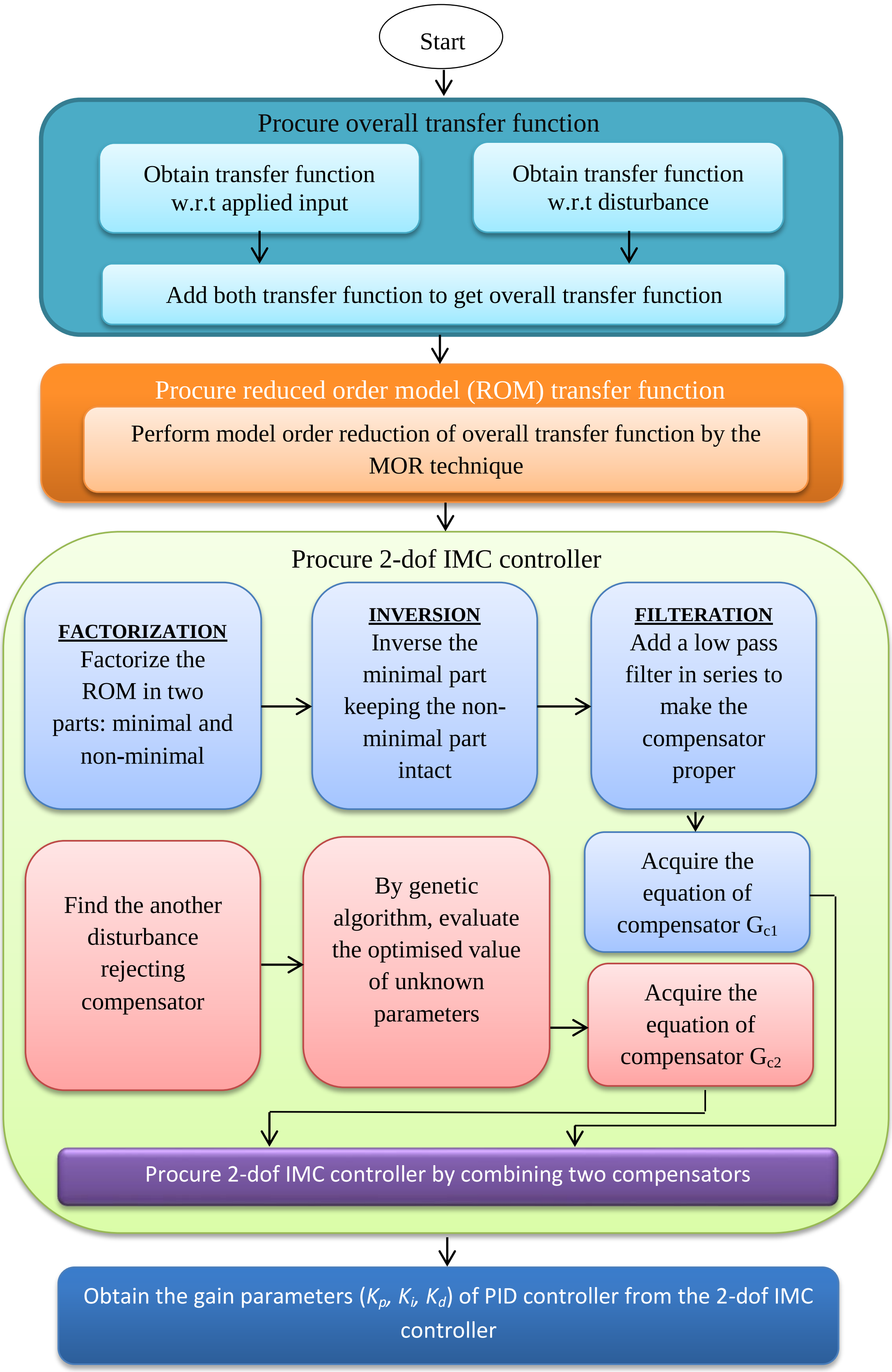

Figure 4.3. Flow chart of the methodology to design 2-DOF IMC-PID controller

$$\left|1 - G_r(s) \cdot Q_{c1}(s) \cdot Q_{c2}(s)\right|_{s=-p_1,-p_2,\cdots,-p_m} = 0 \tag{4.26}$$

e) Obtaining 2-DOF controller

The IMC compensator obtained in equation (4.23) and disturbance rejection controller obtained in equation (4.25) is combined to form a 2-DOF controller, as shown in equation (4.27).

$$C(s) = \frac{Q_{c1}(s) \cdot Q_{c2}(s)}{1 - G_r(s) \cdot Q_{c1}(s) \cdot Q_{c2}(s)} \tag{4.27}$$

f) Obtaining 2-DOF IMC PID controller

The 2-DOF controller obtained in equation (4.27) is compared with the ideal PID controller having three gain parameters: K_p, K_i, K_d. Then after comparing the coefficients, the value of three gain parameters is obtained. Consider the equation for the controller is written as follows:

$$C(s) = \frac{s^m + u_{m-1}s^{m-1} + u_{m-2}s^{m-2} + \cdots\cdots + u_1 s + u_0}{v_m s^m + v_{m-1}s^{m-1} + v_{m-2}s^{m-2} + \cdots\cdots + v_1 s^m + v_0} \tag{4.28}$$

And the equation of ideal PID controller is written as:

$$G_{PID}(s) = K_p + \frac{K_i}{s} + K_d \cdot s \tag{4.29}$$

$$\begin{bmatrix} v_0 & 0 & 0 \\ v_1 & v_0 & 0 \\ v_2 & v_1 & v_0 \\ \vdots & \vdots & \vdots \\ v_m & v_{m-1} & v_{m-2} \\ 0 & v_m & v_{m-1} \\ 0 & 0 & v_m \end{bmatrix} \cdot \begin{bmatrix} |K_i| \\ |K_p| \\ |K_d| \end{bmatrix} = \begin{bmatrix} 0 \\ u_0 \\ u_1 \\ \vdots \\ u_{m-1} \\ 1 \\ 0 \end{bmatrix} \tag{4.30}$$

Hence, the equation of controller derived in equation (4.28) is then compared with the ideal PID controller of equation (4.29), and the value of gain parameters is obtained by the least-square model matching method as described in equation (4.30). So, the value of gain parameters obtained from equation (4.30) represents the 2-DOF

IMC-PID controller. The brief procedure to procure the two degrees of freedom PID controller employing IMC is understood from figure 4.3.