# CHAPTER 1
# INTRODUCTION

The complexity of control mechanisms is increasing every day, and hence the need to form novel techniques of reducing the order of a model is increasing every day. A sound approximated reduction of the system of a large order is generated by employing model order reduction techniques. In the world, many pieces of research are taking place in order reduction of a model to approximate the system of large order into an approximated system of reduced-order through the minimization of the error amongst the two systems, hence increasing the accuracy of the designed system of reduced order. The reduction in the order of a model is used in many disciplines with different interpretations. The techniques based on mathematical approximations for large differential equations are utilised to achieve the small-scale approximation from this large-scale system. In other words, the concept of MOR is associated with mathematical terms such as “dimensionality reduction”, “high energy dynamic models”, “state truncation”, “reduced-bases approximation”, and “balancing the grammians”. The complete concept is generated initially from the mathematics associated with differential equations. Furthermore, due to the comprehensive feasibility of the differential equations concept, the idea of MOR is further implemented in control systems and then enhanced to other fields like civil engineering, automobile engineering, aerospace engineering, Micro-Electro-Mechanical-Systems (MEMS), mechanical engineering, VLSI circuit designing, geological engineering, and many more.

This chapter includes the introduction of the topic and fundamental concepts which are related to the thesis.

## 1.1. GENERAL IDEA ABOUT MODEL ORDER REDUCTION

The universe is made of matter and energy, as shown in figure 1.1. Engineering science works to understand these two states and develop the models and systems available in the universe (Mohamed, 2018). The set of mathematical equations describes the physical phenomenon for these systems. These equations are named Partial Differential Equations (PDE), further discretized to obtain Ordinary Differential Equations (ODE). These ODEs approximates the original PDEs model, whose dimensions are governed by the spatial mesh size. Hence, the finer mesh indicates that the dimension of ODE is large. The number of variables and dimensions of the ODE can be extended from hundreds to millions, depending on the original PDE and mesh size. Then, the large system constituted by ODE is integrated in time to get the feasible solution of the system. The time-domain response of these large systems will require great computational effort. These computational complexities are reduced by model order reduction. Figure 1.2 explains the process more clearly.

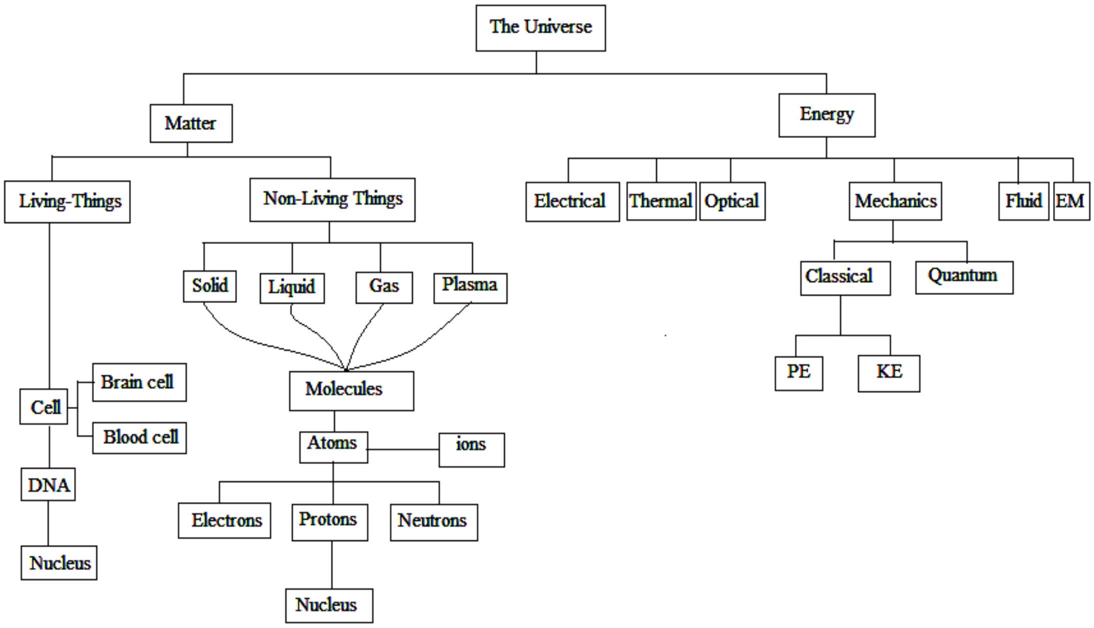


**Figure 1.1. Big picture of the universe**

Considering the case of Maxwell's equations applied in the transmission line. The equations for transmission line (PDEs, Maxwell's equations) can be attained by discretising the line of transmission into small infinitesimal sections; consisting of combinations of resistance, capacitance and inductance and conductance at small intervals the line. These small segments are termed ODEs based on Kirchhoff's law and are combined to form the large order PDE based on Maxwell's equations. The ODEs can be solved efficiently (from Kirchhoff's law) with fewer computations and results in the solution of transmission lines (PDEs).

In the model order reduction process, initially, the *'m'* ODEs that is obtained by discretization of large PDEs and can be described as

$$\frac{dy}{dt} = f(y,t), \qquad y \in \Re^m \qquad (1.1)$$

The large number *'m'* ODEs, i.e. equation (1.1), is approximated into a smaller number *'n'* ODEs by reducing its order while preserving all essential characteristics of the system of *'m'* order as shown in equation (1.2). It satisfies the condition that the number of states *'n'* in the reduced system must be lesser than the states *'m'* in the large-scale system. In other words, the number of differential equations in the reduced system of equation (1.2) must be much lesser than that in the original system in equation (1.1), which can be depicted from equation (1.3).

$$\frac{dz}{dt} = f(z,t), \qquad z \in \Re^n \qquad (1.2)$$

$$n << m \qquad (1.3)$$

Hence, to obtain a good understanding of the large scale system and get a feasible and easy solution of these systems in lesser time intervals, the concept of model order reduction has emerged as an essential tool. The different inventions in the field of reduction of model order have made the idea more interesting. Hence, it becomes prevalent in real-life applications in numerous branches of science.

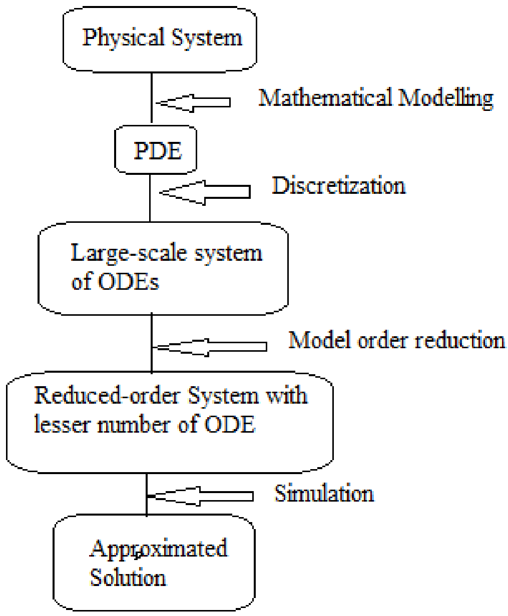


**Figure 1.2. Process of model order reduction**

## 1.2. KEY FEATURES OF MODEL ORDER REDUCTION

While reducing a system of higher-order to a smaller approximation, it is necessary to ensure that the MOR technique is valid or not. Various requirements are to be satisfied and are outlined as follows:

- **Stability.** The reduction technique must be capable of obtaining the reduced model which is stable. The reduced system must have the same stability conditions as possessed by the system of large order.
- **Characteristics preservation.** The system of reduced-order must possess the same properties, which are present in the original system. These properties may be performance characteristics of time-domain like rising time, overshoot, settling time, steady-state error and performance characteristics in frequency domain like gain margin and phase margin.
- **Accuracy.** The reduced-order system obtained by the MOR technique must be accurate with minor error and the same behavioural characteristics as that of the original model.
- **Computational efficiency.** The approximated system must have a low computational cost compared to the original system to fulfill the aim of model order reduction. It should have a low simulation cost and should consume less memory in the CPU.
- **Universal.** The reduction method must be repeatable to all the large-order systems. That means it should be applicable at any place, time and condition.

These requirements guarantee that the reduced model is a replica of the large-scale system and can replace the large-scale system. So, if a MOR technique fulfills all these requirements, then the method helps obtain the approximation of reduced order of the system of large order.

## 1.3. NEED OF MODEL ORDER REDUCTION

The reduction in the order of a system is becoming a valuable tool for studying the performance of large-scale systems. The system’s study becomes more manageable once it gets reduced, as the complications in the study of the system get reduced by reduction in the order. Model order reduction process is essential for the control system for the following reasons:

- A large-scale system can be analysed easily after converting it into a small-scale model with the same characteristics as the higher-order system (J. Singh et al., 2016).
- The model of reduced-order helps to understand the behaviour of a dynamical system better and helps to control the behaviour of the original system for attaining desired performance characteristics (Imran & Ghafoor, 2015).
- The complexity of the large-order system is reduced by approximating the original system into its lower order approximation (O. Alsmadi et al., 2019).
- The reduction in the order of a system helps simulate the system at a faster rate (Sakamoto et al., 2020).
- The computational time of an electrical circuit can be reduced after converting it into its equivalent lower-order form (Sato & Igarashi, 2016).
- The simulation of high-speed circuits and large space structures becomes easy after reducing them into the equivalent lower-order circuit (Gallivan, 1996).
- The dimensionality of simulation of a large system requires large storage space, and this is overcome by using the model order reduction (Antoulas et al., 2001), which uses less space instead.
- The process of model order reduction makes the large-scale systems' future advances comparatively easier (Cheng & Scherpen, 2018).

## 1.4. PERFORMANCE CHARACTERISTICS OF PHYSICAL SYSTEM

A physical system has various characteristics which make it suitable for operation. If a physical system does not have the desired value of these performance characteristics, then that system is not fit to be employed for each input and does not give the desired output. Some critical performance characteristics are described as follows:

### 1.4.1. Time-domain performance characteristics

The time-domain performance characteristics of a system depend on the system's behaviour in the time domain. These characteristics are specified in transient behaviour and steady-state behaviour. The time taken by the system to reach its final value is termed as transient response, whereas after getting the absolute value, the steady-state response is obtained. Both types of behaviour of the system are necessary to check the effectiveness of the system. The time response characteristics of the system lying in a transient state and steady-state are categorized as follows:

(i) **Rise Time.** It is the time taken by the system's response to deviate from an exact low value to an actual excessive value. These values can be expressed as ratios or, equivalently, as probabilities with admiration to a given reference value. Those probabilities are usually the 10% and 90% (or equivalently 0.1 and 0.9) of the output step height.

(ii) **Overshoot.** Overshoot is the presentation of a signal which exceeds its target value. In control systems, overshoot is the value of response when it exceeds its steady-state value. In unit step input, the overshoot is computed as the obtained maximum value minus the steady-state value divided by the steady-state value. The first overshoot of the system is taken as maximum overshoot and is used in the performance analysis.

(iii) **Settling Time.** It is the time needed for the reaction to arrive at a consistent state and stay inside the predetermined resistance groups around the last worth. As a rule, the resistance groups are 2% and 5%.

(iv) **Steady State Error.** The difference between the response of a given system and obtained system in the constant domain is computed as a steady-state error. This error should be ‘zero’ for better reduced-order approximation.

(v) **Integral Square Error (ISE).** The ISE detects the minor error between the higher and reduced systems because ISE takes the sum of the square of error, so a small error gets squared and compares the two systems. This error is represented by equation (1.4).

$$ISE = \int \left[ y(t) - y_r(t) \right]^2 dt \qquad (1.4)$$

(vi) **Integral Time Absolute Error (ITAE).** The error existing between the given higher-order system and obtained reduced approximation is multiplied by time before taking its sum. This error is preferable to detect the error which takes place after long time intervals. This error is computed in equation (1.5).

$$ITAE = \int t.\left|y(t) - y_r(t)\right|dt \tag{1.5}$$

(vii) **Integral Absolute Error (IAE).** This error is obtained by simply finding the difference between two outputs, i.e. higher-order and reduced-order systems. This error computes the absolute value of the difference in the two systems and is represented by equation (1.6)

$$IAE = \int \left|y(t) - y_r(t)\right|dt \tag{1.6}$$

The desired output of the system is contained in y(t), whereas the obtained result is denoted by $y_r(t)$ in the equations (1.4-1.6).

### 1.4.2. Frequency domain performance characteristics

In the frequency domain, the system's performance is estimated by the parameters that exist in the frequency domain. The frequency-domain characteristics depend on the system's magnitude response and phase response for the applied input. These frequency domain characteristics are classified as follows:

(i) **Gain crossover frequency.** The frequency obtained from the magnitude response at which the system's gain is unity or zero dB is termed gain crossover frequency.

(ii) **Phase crossover frequency.** The frequency obtained from the phase response at which the value of the phase becomes $-180^o$ is termed as phase crossover frequency.

(iii) **Gain Margin.** The value of gain over the unity at phase crossover frequency is termed the system's gain margin.

(iv) **Phase Margin.** The value of phase that deviated from $-180^o$ at gain crossover frequency is termed the system's phase margin.

The value of GM and PM also defines the stability of the system. For a system to be stable, the value of GM and PM should be positive. So stability is also termed as a major area of focus.

The parameters procured by time-domain characteristics and frequency domain characteristics determine the reduction algorithm's effectiveness. Suppose the system of the reduced-order system is analysed as approximately similar to that of the system of large order. In that case, the system of reduced-order can be used to replace the large-order system.

## 1.5. REPRESENTATION OF LINEAR & DYNAMIC SYSTEMS

A system can be represented either in the transfer function representation that comprises a fraction of two polynomials or by the state-space model representation having a combination of state matrices. These two forms are described as follows:

### 1.5.1. Transfer function representation

A system is represented in transfer function form that shows the relation amongst the input signal and output signal in Laplace, i.e. *'s'* domain. The transfer function of the system models the output of the system for each possible input applied to it. The transfer function polynomial is represented as follows:

$$G(s) = \frac{Y(s)}{X(s)} = \frac{N(s)}{D(s)} = \frac{b_0 + b_1 s + b_2 s^2 + b_3 s^3 + \cdots\cdots + b_m s^m}{a_0 + a_1 s + a_2 s^2 + \cdots\cdots + a_n s^n} \qquad (1.7)$$

The coefficients *'a' & 'b'* describe the behaviour of the system. From equation (1.7), it is clear that *G(s)* is the transfer function of a system, which is a combination of output obtained *Y(s)* and input *X(s)* applied to the system. The numerator *N(s)* has roots called 'zeros' of the system, whereas the denominator D(s) has roots termed 'poles'. The poles of the system define the stability of the system (Paul et al., 2017). In the system shown by equation (1.7), the number of zeros is *m*, whereas the number of poles is *n*. The system transfer function must be chosen in a manner such that $m<n$.

### 1.5.2. State space representation

Another way representing the behaviour of the system is a state-space model with the use of the state variable of the system. The complete system behaviour is presented by four matrices *A, B, C & D* and having state variables as represented by equation (1.8 – 1.9) as follows:

$$\bar{x} = A \cdot x + B \cdot U \tag{1.8}$$

$$y = C \cdot x + D \cdot U \tag{1.9}$$

The state transition matrices are denoted as *A*, *B*, *C* and *D* and state variables *x* represents the system's state. An output represented by *y* is obtained while applying the input *U*. The two equations (1.8) and (1.9) describe the behaviour of the system's state depending on the state variables; hence the equations (1.8-1.9) are termed as a state-space model of the system. The solution of these two equations describes the behaviour of the system.

## 1.6. EXISTING METHODS OF MODEL ORDER REDUCTION

Since the year 1970, numerous techniques are developed for reducing the model of a system. The MOR techniques apply to all systems, whether represented in transfer function or state-space model representation (Ankur Gupta & Manocha, 2019). The existing methods of model order reduction, which are applicable to reduce the system of a large order, are described as follows:

### 1.6.1. Continued fraction expansion

Continued fraction expansion method is the simple mathematical technique to divide numerator equation by denominator equation. This mathematical technique is extended for model order reduction by (Shamash, 1974). Consider that a physical system is represented in its transfer function form by the equation (1.10), which has a numerator and a denominator as shown in equation (1.10):

$$G(s) = \frac{N(s)}{D(s)} = \sum_{i=0}^{m}\sum_{j=0}^{n}\frac{b_i s^i}{a_j s^j}, \textit{ such that } m>n\ () \quad (1.10)$$

A long division process converts this equation into the cauer form. From this cauer form of an equation, the upper states are truncated by keeping the lower 'k' states intact (where 'k' is the lower-order system order). After the truncation process, the system gets reduced to the 'k' order cauer from structure, converted back to transfer function form by the reverse operation. So, the higher-order system of 'n' order is converted to 'k' order reduced-order system by continued fraction expansion. The technique applies to a single variable and a multivariable linear system (Lucas, 1986).

### 1.6.2. Moment matching algorithm

The technique of the moment matching algorithm is also a mathematical computation technique utilised by (Lal & Mitra, 1974) for the process of model order reduction. In this technique, a moment of the original system is obtained by expanding the equation (1.10) in positive power series as shown in equation (1.11).

$$G(s) = \sum_{x=0}^{\infty} c_x s^x \quad (1.11)$$

The constants '$c_x$' is related to the moments of the system.

Similarly, the transfer function of the system of reduced-order is obtained by unknown coefficients. The moments of the higher-order system are matched with the moments of a system of reduced order. The result gives unknown coefficients of the reduced-order system (Prajapati & Prasad, 2020a).

### 1.6.3. Padé approximation

The time-moments and Markov's parameters of a system are the two factors that help to obtain the Padé approximation of the system transfer function. The padé approximation technique works according to Koenig's theorem (Shamash, 1975). This technique states that a rational function of two polynomials, $P$ (degree $i$) and $Q$

(degree *j*), is procured, such that the multinomial $\left[P_i(x)\right]/\left[Q_j(x)\right]$ is a Padé approximant. This polynomial form obtained for a function *f(x)* is a Padé approximant if the power series expansion of the rational function $\left[P_i(x)\right]/\left[Q_j(x)\right]$ and *f(x)* is identical for all the terms.

Considering that the function *f(x)* is obtained for the system of a large order is obtained by dividing the numerator equation by its denominator equation as described in equation (1.12) as follows:

Let, $f(x) = c_0 + c_1 s + c_2 s^2 + \cdots\cdots,$ *such that* $c_i$ *is real and* $c_0 \neq 0$ (1.12)

Similarly, a rational function having two polynomials P and Q is obtained from the reduced-order transfer function as follows:

$$\frac{P_i(x)}{Q_j(x)} = \frac{e_0 + e_1 s + e_2 s^2 + \cdots + e_i s^i}{f_o + f_1 s + f_2 s^2 + \cdots + f_j s^j} \quad (1.13)$$

The equation (1.13) and power series expansion of equation (1.12) are compared, and the Padé approximant coefficients are obtained. This technique applies to single and multivariable systems (Bandyopadhyay B. and Lamba S.S, 1987; Xiheng, 1987). The modification of the Padé approximation is presented later on (Dewangan et al., 2020).

### 1.6.4. Stability equation method

Another technique of model order reduction applicable for reducing the transfer function is the stability equation method (Chen et al., 1979). In this technique, the even and odd parts of the transfer function of equation (1.14) are separated into the even and odd polynomials.

$$G(s) = \frac{N_e(s) + N_o(s)}{D_e(s) + D_o(s)} \quad (1.14)$$

The even and odd parts, i.e. the terms denoted by subscript ‘e’ and subscript ‘o’ respectively, are termed stability equations. From the pole-zero pattern, the poles farther from the origin and are of less significance are rejected. The poles of higher importance are selected from the stability equations such that the order of the G(s) becomes equal to the order of the system of reduced order.

### 1.6.5. Factor division method

The long computations of the model order reduction techniques are reduced by the factor division method. This technique avoids calculating time moments and Markov's parameters by simply dividing out the equation (1.10) of the large-scale system by the unwanted pole factors (Chen et al., 1980; Lucas, 1983). This technique is based on the principle of dominant pole retention. The denominator of equation (1.10) is formed of the factors of poles of the system and is clear from equation (1.15):

$$G(s) = \frac{b_0 + b_1 s + b_2 s^2 + \cdots\cdots + b_m s^m}{(s+p_1)(s+p_2)\cdots(s+p_n)} \tag{1.15}$$

The equation (1.15) is then solved by dividing its numerator multinomial from all the factors consisting of the unwanted poles (poles of less importance) and is reduced to procure the new equation having the desired order.

### 1.6.6. Balanced truncation technique

The first technique applied to reduce the state-space model of a large-scale system is the balanced truncation technique. The technique is based on Kalman's theory (Moore, 1981) of minimal realization. The geometric objects (controllability and observability) are balanced here such that the things leading to structural instability get removed from the system. The controllability (*P*) and observability (*Q*) grammians of the system of higher-order as given in equations (1.8-1.9) are obtained as follows:

$$P = \int_0^\infty e^{At} \cdot B \cdot B^T \cdot e^{A^T t} \cdot dt \tag{1.16}$$

$$Q = \int_0^\infty e^{A^T t} \cdot C^T \cdot C \cdot e^{At} \cdot dt \tag{1.17}$$

The controllability and observability grammians from equations (1.16) & (1.17) are obtained to satisfy the Lyapunov's equations (1.18-1.19)

$$A \cdot P + P \cdot A^T = -B \cdot B^T \tag{1.18}$$

$$Q \cdot A + A^T \cdot Q = -C^T \cdot C \tag{1.19}$$

The grammians are identical and give the diagonal matrix truncated by eliminating the states of less importance and keeping the *'r'* states equal to the order of the system of reduced order. By reverse action, the matrices *P & Q* are obtained and hence the matrices of the reduced-order, i.e. *A', B' C'* and *D'*, are obtained. The MOR based on the balanced truncation technique is equally applicable to continuous-time and discrete-time systems (Pernebo & Silverman, 1982).

## 1.6.7. Hankel norm approximation

The technique for reducing the model order of the linear system of a single variable and multivariable applicable to the model presented in state-space form is Hankel norm approximation. This technique involves the computation of the norm of error $\|G(s) - G_r(s)\|$, which is to be minimized (Glover, 1984). $G_r(s)$ is the reduced-order approximated system of order *r*. The controllability and observability grammians are obtained from equations (1.16) & (1.17), and the Hankel norm can be obtained by equation (1.20).

$$\|G(s)\|_H = (\lambda_{\max}(PQ))^{1/2} \tag{1.20}$$

The most controllable and observable states of the system are represented by the value of $\lambda_{\max}(PQ)$ and are obtained from the multiplication of controllability grammian (P) with the observability grammian (Q). The most significant eigen value of matrix PQ gives the Hankel norm of the system. The Hankel norm of the system provides the most actual states, and thus the least significant states can be discarded, depending upon the order of the reduced-order system.

## 1.6.8. Balanced stochastic technique

The balanced stochastic technique represents the canonical correlation coefficient matrix $\sum$ of a system which equates the state covariance matrix for the process at the output terminal (U. B. Desai & Pal, 1984; Shaker, 2008). The spectral factors are obtained in this process, which specifies the frequency characteristics of the system. To approximate the system of reduced-order by BST following steps are to be performed:

- The representation of forward innovations (FIR) and backward innovations (BIR) for the system of higher-order is obtained.
- Ricatti equations are then used to obtain (Enns, 1984; Green, 1988) the correlation coefficient matrix from FIR and BIR.
- The correlation coefficient matrix is then reduced up to the order of the system of reduced-order by truncating the less significant coefficients.
- Reverse transformation is applied to obtain a reduced correlation matrix, and then the forward and backward balanced stochastic realization is procured.

The resulted balanced stochastic realizations represent the transfer function of the reduced system fetched from the system of large scale/ high order by using BST realization.

## 1.6.9. Projection technique

The projection of the system of large scale is made on the reduced approximated system by using the projection technique. The matrices *A', B', C'* and *D'* are the state matrices of the system of reduced-order and are obtained by the projection matrices as given in equation (1.21)

$$(A',B',C',D')=(LAT,LB,CT,D) \qquad (1.21)$$

In equation (1.21), the symbols *'L'* and *'T'* are termed projector matrices. At selected frequencies, the frequency response and their derivatives of the system of approximated reduced order and the system of given high order are matched in the projection technique. Moreover, at these frequencies, the power spectral density of the matrices and some derivatives of it are also compared. These frequencies include various low, medium and high-frequency ranges. At these frequencies, the moments of power and frequency are calculated, and hence the system of reduced-order is obtained (De Villemagne & Skelton, 1987; Hernández-Becerro et al., 2021; Salimbahrami & Lohmann, 2002).

## 1.6.10. Impulse energy approximation technique

The impulse energy of the system can also be utilised for the process of model order reduction (Lucas, 1988). The parameters ($\alpha_i$, $\beta_i$) are required to be calculated from the reciprocating transfer function *H(s)*, such that $H(s) = (1/s) \cdot G(1/s)$, $i=1,2,3,.....n$, *G(s)* represents the transfer function representation of the system of higher-order given by equation (1.22).

$$H(s) = \frac{1}{1 + \alpha_1 s + \frac{1}{\alpha_2 s + \cdots}} \left[ \beta_1 + \frac{1}{\alpha_2 s + \frac{1}{\alpha_3 s + \cdots}} [\beta_2 + \cdots] \right] \qquad (1.22)$$

The value of energy parameters is thus obtained from the ($\alpha_i$, $\beta_i$) pair as $e_i = \beta_i^2 / \alpha_i$ and then 'r' parameter pairs having the most considerable energy are kept, and other fewer energy states are removed. After that, the ($\alpha_i$, $\beta_i$) pair of high energy forms the reduced system.

## 1.6.11. Schur decomposition

The Schur decomposition technique reduces the large-scale system without applying any balancing operation. The matrix obtained by multiplying *P & Q* matrices, i.e. *PQ* matrix is divided into left and right eigen spaces, and then by applying the decomposition of these two eigen matrices, the state matrices of the model of the reduced-order are computed (Aldhaheri, 1991; Huu et al., 2013; Safonov & Chiang, 1989). Following steps are employed in the Schur decomposition:

- The schur equivalent form of the matrix *PQ* is obtained by employing an upper triangular matrix *V* and forming equation $VPQV^T$.

$$V_A^T PQV_A = \begin{bmatrix} \lambda_1 & \cdots & \cdots & \cdots & \cdots \\ 0 & \lambda_2 & \cdots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \lambda_n \end{bmatrix} \qquad (1.23)$$

$$V_D{}^T PQV_D = \begin{bmatrix} \lambda_n & \cdots & \cdots & \cdots & \cdots \\ 0 & \lambda_{n-1} & \cdots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \lambda_1 \end{bmatrix} \tag{1.24}$$

- Then the matrix $V$ is transformed and arranged into its ascending and descending order matrices as shown in equation (1.23) & equation (1.24):
- Now $V_A$ and $V_D$ obtained from equations (1.23) & (1.24) are partitioned in the small and big values based on the order of the system of reduced-order ($r$), and represented in the form as follows:

$$V_A = \left\{ \overset{n-r}{V_{R(small)}} \middle| \overset{r}{V_{L(big)}} \right\} \tag{1.25}$$

$$V_D = \left\{ \overset{r}{V_{R(big)}} \middle| \overset{n-r}{V_{L(small)}} \right\} \tag{1.26}$$

- Then '$r$' significant values $\left(V_{R(big)} and V_{L(big)}\right)$ obtained from equations (1.25) & (1.26) are combined to fetch the state model form of the system of reduced order.

Hence, these steps result in the formation of the model of reduced-order from the decomposition of the high order model.

### 1.6.12. Least square method

This technique is executed to derive the system of reduced-order by minimizing the sum of mean square error by mathematical computations using a least square algorithm. This technique can be applied in two ways: a) white noise is applied to the input of the system, and cross-correlation in the input and output is obtained. Similarly, the auto correlation of production is calculated. Then the mean square error amongst the auto correlation function and cross-correlation is calculated by a least square algorithm (R. J. Lalonde et al., 1992). b) the Second process involves least square matching of the impulse response of the system of large scale and the system of reduced scale. The response obtained from both the processes is the same (Rick J. Lalonde et al., 1993).

### 1.6.13. Routh approximation technique

The concept of Routh approximation is utilised to calculate the polynomial of the denominator of the system of reduced order. The state matrices of the model of reduced-order, i.e. (*A', B', C'* and *D'*) are obtained after time moments/ Markov's parameter matching (Prasad, 2000). The procedure to procure the approximated reduced-order by routh approximation technique is defined as follows:

- Obtain the reciprocal form of the denominator $D\left(\frac{1}{s}\right)$ from the rational transfer function as shown in the equation (1.10).
- Form the routh array from the reciprocal of the denominator polynomial obtained in the first step. The first column of the routh array is taken, and the first column's coefficients derive the denominator of the reduced-order system as shown in equation (1.27).

$$D_r(s) = \cfrac{1}{1+\alpha_1 \cfrac{1}{s} + \cfrac{1}{\alpha_2 \cfrac{1}{s} + \cfrac{1}{\alpha_3 \cfrac{1}{s} \ddots \cfrac{1}{\alpha_r \cfrac{1}{s}}}}} \qquad (1.27)$$

  The coefficients of the first column are represented by the parameter *'α'* of Equation (1.27). The first *'r'* values of the coefficients are taken to find *'α'* parameters of reduced-order denominator polynomial.

- By finding the time moment and Markov's parameter of the system of large scale and comparing them with the reduced order, the state-space model of the system of reduced-order is formed.

### 1.6.14. Response matching technique

In this technique, the response of the system of reduced-order is matched with the response of the original system of higher-order. Consider that a system having twelve poles is to be reduced, and a system of 3$^{rd}$ order is to be formed. These three poles of the reduced-order system are selected randomly from the twelve poles. The response of the higher and reduced systems is matched to get the better combination of three poles that provides a more accurate approximated reduced order (Satakshi et al., 2005). The transfer function of a system can be assumed to have real, complex and repeated poles. Different combinations of poles can be present in a system: (i) system has all real poles (ii) system has real poles, as well as complex conjugate pair (iii) poles, are real, out of which some poles are repeated poles (iv) all poles in the system are repeated. Based on these categories, the system of the reduced systems can be designed randomly and compared to the associated higher-order system.

## 1.6.15. Soft computing based model order reduction

The evolution of techniques based on soft computing provides an accurate approximated reduced order with optimised output. There are numerous soft computing methods developed and applied to achieve the reduction in the order of a system. There are two categories of soft computing techniques, which are described as follows:

### 1.6.15.1. Population-based solution soft computing approach

*a) Invasive weed optimisation (IWO):* IWO is an optimisation technique that is numerically inspired by the colonization of weeds. IWO technique was developed by (Mehrabian & Lucas, 2006). The process of maximizing the number of plants grown is known as IWO. In this technique, an area to grow plants is selected, and it is sowed with seeds. The seeds germinate and grow into a plant. It blooms with flowers that have seeds on it. Consequently, these seeds are collected and sowed again. The same process is repeated, and the seeds are collected again. This collection and sowing of seeds continue till the maximum number of plants has grown in the area.

The process starts with the population, which is initialized randomly and the seeds for the population members are produced based on their relative fitness. These seeds are then scattered randomly on the entire search space by finding the mean and standard deviation of the seeds. Consequently, the seeds produced and their parents are the solution for the next generation. The process continues until the best fitness survivor is obtained and all seeds become reproductive.

*b)* *Particle Swarm Optimisation (PSO):* The technique of PSO is based on obtaining the best fitness function using multiple displacements over a while. The combination of '*N'* particles, called swarm, is placed in a *'D'* dimensional space with a unique topology. The same can be compared with the movement of birds and fishes. Birds keep changing their position to obtain the best possible combination to have the optimised performance to reach the destination. The topologies in the technique of PSO are of two types: one is global best (*gbest*), and another is local best (*lbest*). A swarm must be capable of attaining the best in both the topological networks (Kennedy, J., & Eberhart, 1995; Lavania & Nagaria, 2016).

In the first phase, each particle is initialized by two factors: position and velocity. After each iterative step, every particle changes its velocity and attains a new position in the *D* dimensional space. The change in velocity and position is done to obtain the best possible outcome. These particles keep changing their position in each iteration to reach the destination with the best fitness.

*c)* *Genetic Algorithm (GA):* Genetic algorithm (GA) is a technique that works on the principle of genetics and natural selection and has the foundation on the living being's biological evolution. It deals with space shortage for individuals and attempts to find out the fit individuals by generating the new population iteratively (Vishwakarma & Prasad, 2009). A GA examines a populace of high-quality individuals to a populace of initial individuals, and here, every single populace provides an answer to the problem. The quality of the objective function is calculated using a fitness function, which shows the quantitative representation of the adaptation of the objective function on a particular environment. Moreover, the randomly generated individuals act as the initial population and help initiate the process. The steps involved in the process of genetic algorithm are population initialization, evaluation, selection, crossover, mutation, replacement, and termination. In reducing the error amongst the system of higher-order and corresponding system of reduced order, the error value has to be chosen as the objective function of the genetic algorithm. So, optimisation of objective function means reducing the error (Adamou-Mitiche & Mitiche, 2017).

d) *Cuckoo Search Algorithm:* The process of optimisation is based on the breeding of cuckoo, which is based on probability. Here, a cuckoo searches for a nest to lay eggs. The nest search is carried for the maximum and the best breed; when the cuckoo searches for a nest, it may lay eggs in an empty nest or vacate the nest by disposing of the already present eggs. Now, the nest owner would sit on the eggs and provide heat to them for hatching. The chicken from the egg usually imitates the cuckoo, which has heated the eggs to obtain maximum food and is healthy. On the contrary, the nest owner cuckoo may dispose of the initial cuckoo eggs or build another nest. In this way, the cuckoo eggs do not breed at all. So there is a probability that the cuckoo's eggs hatch or remain wholly isolated and un-hatched. Therefore, it depends on the nest owner cuckoo whether the hatching takes place or not. The role of the initial cuckoo is just limited to find such a nest where the eggs can bread adequately, which would, in turn, provides the maximum chances of better breeding (Yang & Deb, 2009).

*e)* *Firefly Algorithm (FA):* The way followed by fireflies for obtaining the optimised solution of a problem forms the basis of the optimisation process. The firefly algorithm of soft computing is an algorithm used to find the optimal solution and features of a given problem. Here, fireflies keep on changing their position and brightness in a predefined area to obtain the optimised function. The position of the firefly indicates the optimised solution, and its brightness indicates features. Brightness also acts as an indicator of the objective function used for optimisation. The algorithm follows the following steps: location initialization, brightness computation, result ranking and updating, and termination (O. Alsmadi et al., 2019).

The techniques mentioned here utilise the iterative method of finding the optimised solution to the problem.

#### 1.6.15.2. Single-solution based optimisation

*a)* *Artificial Neural Network (ANN):* The technique of ANN is based on the biological neural network. Neurons are the significant part of a biological neural network, and it consists of dendrons and dendrites. The flow of information is through these neurons, and the information exchange takes place at the synapse. Similarly, ANN is based on this transfer of information between the input and output terminals through neurons, where each transfer path has its weight. The ANN is trained initially, and then it possesses a capability similar to the biological neural network. The input and output values are applied to the ANN, and it keeps on changing its weights (training) to obtain the optimised path (Adel & Salah, 2017). The process of ANN also forms the basis of the model order reduction process (Daniel et al., 2020).

*b)* *Fuzzy Logic:* Fuzzy logic is based on the partial probability of occurrence. The probability of an event happening is either true or false (0 or 1), but in fuzzy logic, the value can be between 0 and 1 (A. K. Singh & Purohit, 2014). It has a membership function known as the gain function, which provides the output value based on the input value. Fuzzy logic has wholly transformed the domain of probability (Gautam et al., 2019; Narain et al., 2014; H. Singh et al., 2013).

### 1.6.16. Mixed approach

The evolution in the reduction of the order of a system resulted in a more accurate reduced-order approximation. The error is existing among the system of large-scale and the system of reduced-order has become significantly less. Mixed approaches are developed to increase accuracy and make the model order reduction theory more effective. These mixed approaches are a combination of previous techniques discussed in the preceding sections. A more accurate approximation of the system of higher-order can be obtained by applying two different techniques on two polynomials (numerator and denominator), as shown in figure 1.3. The combination of two algorithms of model order reduction, i.e. routh Hurwitz array and factor division algorithm, was presented by Singh (N. Singh et al., 2006). After that, the mixed approach was presented (Parmar et al., 2007b) to join the two popular techniques, i.e. eigen spectrum analysis and factor division algorithm. The researcher (Philip & Pal, 2010) introduced the frequency-dependent dominant pole retention technique with the big bang big crunch theory. The mixed approaches have gained more popularity because these techniques combine the nature of two different techniques to get higher accuracy by making the system of reduced-order more approximate to higher-order. More combinations of the two techniques were procured later on by various researchers (Gautam et al., 2019; Narwal & Prasad, 2017; Prajapati & Prasad, 2019; C. N. Singh et al., 2019; Tiwari & Kaur, 2020b)

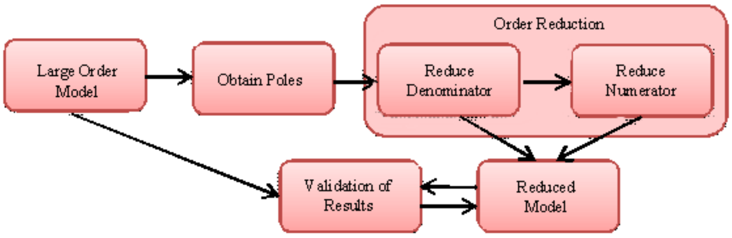


**Figure 1.3. The basic process of a blended approach based MOR**

## 1.7. APPLICATIONS OF MODEL ORDER REDUCTION

Since the evolution of the concept of reduction of order, it is being used widely in science and technology.

- *Biomedical engineering:* The process of reducing the order of a model is used for the surgical simulation of the response of the soft tissues. The viscoelastic problem occurring in the simulation of soft tissues is eliminated by using the technique of model order reduction (BaniHani & De, 2009). The patient-specific design of mechanical models of a human liver became feasible by implementing the model order reduction technique (Lauzeral et al., 2019).
- *Thermal & Mechanical engineering:* The induction heating problem occurred in the cylindrical billets and induction cook top. It is simulated and reduced using the model order reduction technique (Codecasa et al., 2016). The machine tools used in the design of thermo-mechanical systems are focused and implemented quickly by applying the model order reduction technique (Hernández-Becerro et al., 2021).
- *Electromagnetics:* The concept of model order reduction is also gaining popularity in electromagnetic theory and helps in the problems occurring in electric machines. The MOR method is applied to analyse the linear eddy current problem in the quasi-state electromagnetic field having electric and magnetic modes (Kameari et al., 2018).
- *MEMS engineering:* The computation burden of the lengthy finite element models such as Radio-frequency micro-electro-mechanical systems (RF-MEMS) can

be reduced by an efficient model order reduction technique (Rudnyi & Korvink, 2006). This helped in designing the future generation wireless terminals.

- *Cyber-physical system:* Cyber-physical systems are becoming a topic of interest these days due to the additional security measures taken for cyber systems. These cyber-physical systems are large ordered systems, which are required to be reduced for prospects. So, an efficient technique used for reducing the order of a model has been developed now for cyber-physical systems (Song et al., 2015).
- *Electrical engineering:* Model order reduction theory is applied to electrical systems to make their study more accessible. The reduction of passive RLCK circuits by employing the model order reduction technique was done by Yan (Yan et al., 2008). The complexity in obtaining the lumped parameters of a transformer is made easy by using the model order reduction technique (Srinivsan & Krishnan, 2010). The complexity of large power systems can be reduced by applying an efficient model order reduction method and preserving its structure (O. M. K. Alsmadi et al., 2014). The analysis of fast rotating electric machines is feasible using the model order reduction concept (Sakamoto et al., 2020).

## 1.8. BASIC IDEA OF CONTROLLERS

A process model is a simple closed-loop model that can be represented as shown in figure 1.4. It is clear from the figure that the system has two highlighted components: controller and process. There is always an input to the process, denoted by *'R'*, some process variables and then an output *'C'*. The controller part controls the output, which is hard to achieve by using only process variables (Åström & Hägglund, 1995). The desired output of the system is called set value (SV), and obtained output of the system is called process value (PV) (detected from a sensor). The difference between SV and PV is called a control error, i.e. $e = C_{sp} - C$. The error among the set value and process value is to be minimised through the control of the parameters of the controller. So the purpose of implementing a controller is to control the process value so that it can become comparable to the set value by eliminating all the disturbances occurring in the process. The procedure of obtaining the variables of controllers is known as tuning.

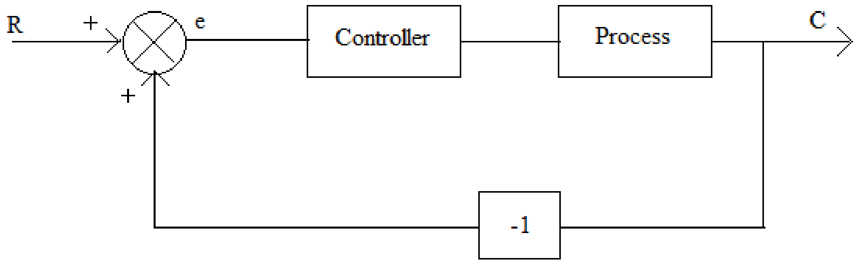


**Figure 1.4. The basic design of a process model**

## 1.9. TYPES OF CONTROLLER

The tuning of the controller is done according to the changes required in the controller's variables to obtain the set value (Astrom & HÄgglund, 2006). Based on these changes, there are various types of controllers described as follows:

*a)* *ON-OFF control:* The control mechanism of a simple ON-OFF controller can be defined from the equation (1.28). The controller acts as a simple on-off switch that goes 'ON' when an error is *positive* and 'OFF' when an error is *negative*.

$$R = \begin{cases} R_{\max}, if e > 0 \\ R_{\min}, if e < 0 \end{cases} \tag{1.28}$$

There are only two modes of the applied input to the process; either it is the highest value or the lowest value. The limitation of this control is that the system can exhibit oscillatory behaviour.

*b)* *Proportional control:* The limitation of ON-OFF control is its oscillatory behaviour, and the proportional control can eliminate it. The slight change in error will produce the change in process output and obtain the set value. In these controllers, the system's output is directly proportional to the control error arising in the control system. The process model with the proportional controller is represented in figure 1.5. The equation (1.29) shows the process of proportional control having 'K' known as the controller gain.

$$R = K_p \cdot e = K_p(C_{sp} - C) \tag{1.29}$$

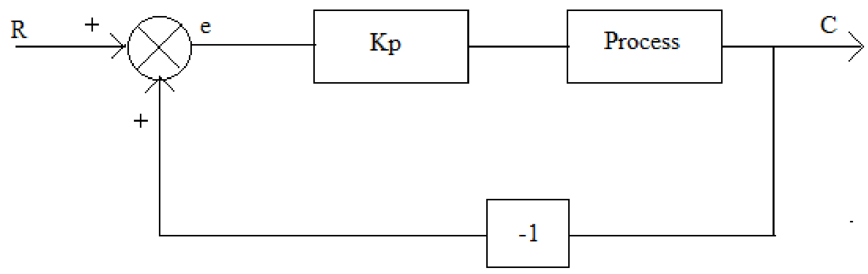


**Figure 1.5. Block diagram representation of process model with a proportional controller**

*c)* *Integral control:* The limitation of proportional control is that the output often deviates from the set value. This limitation is removed by employing the integral type of control. The integral of error is used to obtain the process input *'R'* and is represented in equation (1.30).

$$R(t) = K_i \int e(t).dt \tag{1.30}$$

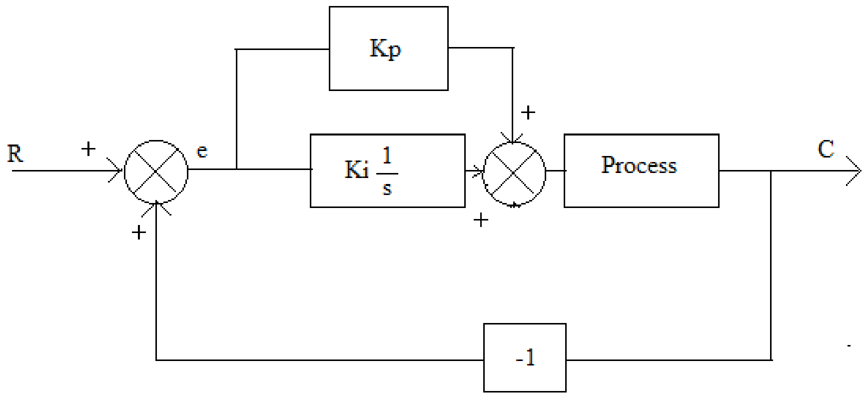


**Figure 1.6. Block diagram representation of process model with proportional integral controller**

The integral controller has the gain parameter $K_i$, as shown in equation (1.30). The proportional-integral (PI) controller is formed by combining the integral control with the proportional control, which possesses the process input as given by equation (1.31) as follows:

$$R(t) = K_p.e(t) + K_i \int e(t).dt \tag{1.31}$$

The process model having a proportional and integral controller combined within is shown in figure 1.6.

*d)* *Derivative control:* Another type of control used for removing the limitations of the proportional control is a derivative control. The derivative of error is used to obtain the process input *'R'* and is displayed in equation (1.32) as follows:

$$R(t) = K_d . \frac{de(t)}{dt} \tag{1.32}$$

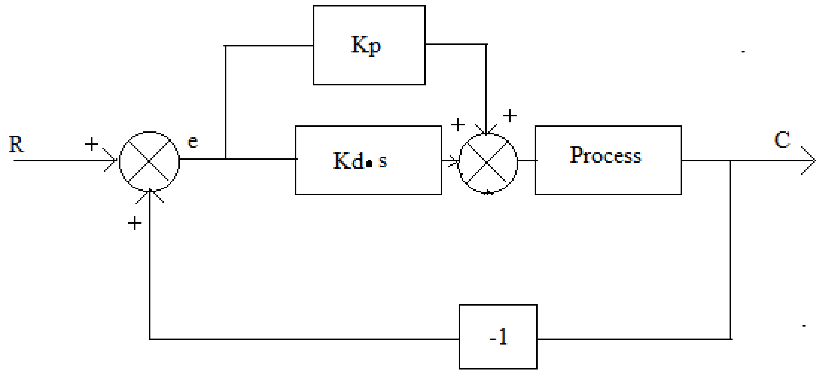


**Figure 1.7. Block diagram representation of process model with proportional derivative controller**

*$K_d$* is the gain parameter of derivative control. The derivative control can be combined with proportional control to form the proportional derivative controller, which is represented by the equation (1.33) as follows:

$$R(t) = K_p . e(t) + K_d . \frac{de(t)}{dt} \tag{1.33}$$

The combination of proportional and derivative control forming a proportional derivative control is presented in figure 1.7

*e)* *PID control:* The design of the PID controller is based on the anticipative nature of the controller by predicting the output based on linear extrapolation. This control is a refinement of the existing proportional, integral and derivative control. The process of PID control can be easily understood by the equation (1.34) derived to obtain the process input, as follows:

$$R(t) = K_p . e(t) + K_i \int e(t).dt + K_d . \frac{de(t)}{dt} \tag{1.34}$$

Thus, the control action taken here is a sum of all three control mechanisms (proportional, integral and derivative). This is also depicted in figure 1.8 by showing the system having a PID controller.

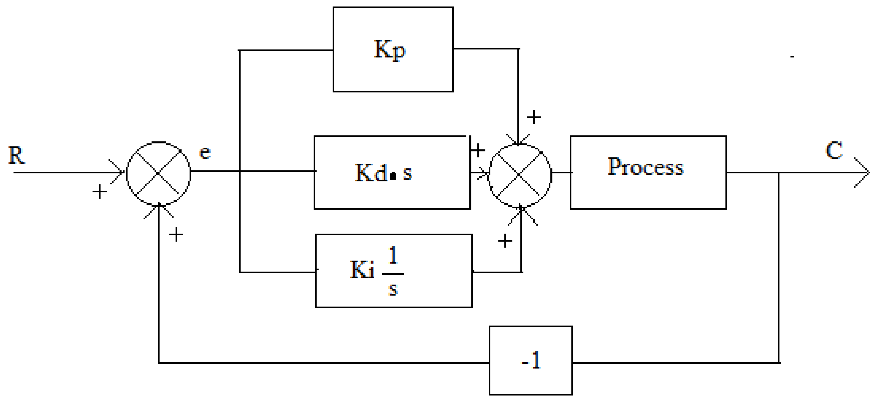


**Figure 1.8. Block diagram representation of process with PID controller**

PID controller has evolved as a significant controller for controlling a wide range of control applications. So, many controllers are designed these days based on the gain parameters of the PID controller. So, the PID controller's study makes the design of controllers easier.

## 1.10. DEGREE OF FREEDOM OF A CONTROLLER

The number of controllers in a control system designed to control the desired parameters of the control system is defined as its degree of freedom. The two degrees of freedom controller exhibits better performance than one degree of freedom controller (Taguchi & Araki, 2000). The number of tuning parameters also describes the degree of freedom, i.e. more freedom means more tunable parameters in the control system. Hence, the more tunable parameters mean a better-controlled environment. The second controller in a two degree of freedom controller can be joined with the first controller in feed-forward or feedback.

## 1.11. LOAD FREQUENCY CONTROL (LFC)

The frequency variation is a significant issue for any power system because the system stability is dependent on the stability in its frequency of operation. By the steady frequency and active power balance, the stability of a system is attained. If there is any change in the active power balance of a power system, the frequency of this system cannot be stable at the rated value (Sharma et al., 2017). Hence, there will be oscillations in power as well as in the frequency behaviour. Thus, there is an instability issue (Bahgaat et al., 2014). In the electrical power generation systems, the changes in the load cause the changes in desired frequency value. These load changes can be because of line faults, line trips, tie-line power changes, or overload. Load frequency control (LFC) or automatic generation control (AGC) is responsible for the system's stability and power output in this load changing environment (Aditi Gupta et al., 2020). The primary purpose of automatic load frequency control is to optimise the actual control error (ACE) (variation in frequency and change in tie-line power) and keep the frequency and power deviations in the desired limits. The role of LFC for the power system is to a) maintain the zero steady-state error despite the deviations in frequency, b) prevent unexpected load disturbances, c) minimize power flow to neighbouring areas, d) manage the system uncertainties and nonlinearities within tolerable range e) ensure the performance under prescribed conditions and parameters (Saxena, 2019; Saxena & Hote, 2013). Several control schemes have been proposed to eliminate the problem of load frequency control and are described as follows:

### 1.11.1. Integral control

The conventional PI control was initially utilised for load frequency control. This controller increases the order of the system and ensures steady-state error at 'zero' while the load is the step input. The controller's gain ($K_i$) is chosen by the hit and trail method for the good shaped transient response. Firstly, routh-hurwitz criteria are applied to the process model to get the range of the gain parameter. Then, the exact value is obtained by the hit and trial method to get the optimum response. This method gives a choice to set the desired value of overshoot, gain and phase margin, and bandwidth of a closed-loop system. The technique was implemented for the LFC problem (Khodabakhshiar & Golbon, 2004).

### 1.11.2. Sliding mode control

The control technique applicable to real power plants in the fast-changing environment is discrete-time sliding mode control (Vrdoljak et al., 2010). The discrete-time is chosen to cope up with the fast-changing characteristics of the system. These changes are analysed in a band of time instead of each instant of time. The steps involved in obtaining the controller are as follows:

(i) Choose a sliding surface on which state of the system is forced to reach and stay on the sliding surface. Hence, the new state matrices are computed, which forces the states to stay on the sliding surface.

(ii) Then a control law is computed to approximate the continuous time into an appropriate discrete-time system for forcing the states to reach the sliding surface.

(iii) Then an optimisation principle, i.e. genetic algorithm, is applied to the system after it achieves a sliding surface that works on the minimization of ACE for a steady-state sliding mode.

The technique of sliding model control applies to thermal as well as hydro power plants. Hence, it is defined as a universal technique of the design of a controller. Many advances of the sliding mode controllers are presented by the researchers (M. Singh & Chandra, 2010).

### 1.11.3. Intelligent control

The use of evolutionary algorithms based on genetic principles and fuzzy logic is used to solve load frequency control. The genetic algorithm was used to reduce load variation on frequency (Rerkpreedapong et al., 2003). The parameters of the conventional PI controller get tuned by the genetic principle. Then the concept of fuzzy logic was used for the control of frequency variation due to load for an electrical power system (Balamurugan, 2018; Revathi & Mohan Kumar, 2020; Shakibjoo, Moradzadeh, Moussavi, et al., 2020). Fuzzy logic damped out all the oscillations produced in the frequency due to variation in load using the fuzzy gain scheduled proportional and integral controller (FGPI). The evolutionary techniques promised a better way to control the frequency variation, and hence intelligent control is obtained (Chopra et al., 2014; Tiwari HP, 2016). The particle swarm optimization technique is also implemented for obtaining the controller (Jagatheesan et al., 2017).

### 1.11.4. Self-tuning and adaptive control

Based on the adaptation of the system parameters, the command on frequency variation is done in adaptive control. These controllers use only the available information about the states and output of the model and the plant for control purposes. The adaptive control can be from two methods: firstly, by self-tuning the parameters and another based on the model reference. The application of a self-tuning regulator is utilised in controlling the frequency variations. The sum of the error occurring in the power system's frequency due to load variation and tie-line power deviation is used to tuning the PI controller, and ACE is minimized. It is done by tracking and minimizing the variances in error occurring in the frequency and power deviation in the tie-line by variation of the weights related to these two variables. The controller based on self-tuning computes the changes in load at a faster rate. The model reference-based adaptive control (Chandra et al., 1988; Pan & Liaw, 1989; Sahu et al., 2016) was applied on the electrical power systems for obtaining the parameters of a PI controller to satisfy the hyper-stability condition for changing the parameters of the plant. The parameter of a reference model is obtained by the pole assignment methodology and is used to estimate the parameters of the PI controller. An advancement in the form of model predictive control is later developed (Sultana et al., 2017)

### 1.11.5. PID control

The conventional PID controller uses a differentiator; therefore, it amplifies high-frequency noise when used in a noisy environment. So these controllers are not suitable to work for the load frequency control applications prevailing in the noisy environment. But the advantages of PID over PI control motivate its use in the control applications. So a low pass filter is added with these controllers, which pass the only low-frequency signal through it and hence neglect high-frequency noises. The PID controllers with low pass filters are therefore helpful for the removal of the LFC problem. Therefore, it is possible to use PID controllers for load frequency control

(Ahmed et al., 2020; Guha et al., 2020; Moon et al., 2001). The optimal PID controller design is presented later on (Noshadi et al., 2016).

### 1.11.6. IP control

IP controller is a combination of proportional and integral control in a new formation other than PI controller. The IP controller is a slight modification of the PI controller as the output of the IP controller varies slowly and has a small magnitude. Also, the bandwidth of the IP controller can be varied depending on the system constraints. The parameters of the IP controller can be obtained by different evolution algorithms such as harmony search algorithm (Boroujeni et al., 2011) and genetic algorithm (Selvakumaran et al., 2014). The performance of IP controllers comes out to be better than conventional PI controllers.

### 1.11.7. Robust control

When the environment is uncertain, it becomes difficult to control the variations by implementing a direct controller. To achieve the goal, robust methods have been developed for the load frequency control problem to eliminate the ACE occurring due to load variations. A robust control mechanism was developed (Ray et al., 1999) using the "matching conditions" and "Lyapunov's stability theory". The control mechanism offers stability to the system. Moreover, the quantitative feedback theory was used as a robust control technique to estimate the controller parameters by adding an uncertain parameter changing environment (Taher et al., 2008). A change of 40% is added in the parameters simultaneously for each iteration.

### 1.11.8. Internal model control (IMC)

The power generation system is formed of power generating units like governors, turbines and generators. This power generating units' value fluctuates every minute based on power flow conditions and system parameters. The controllers designed earlier were not suitable for coping with these fluctuations (Saxena & Hote, 2013). Internal model control emerges as a robust technique to work in such an uncertain environment and handle the fluctuations. The internal model control phenomenon works on the process if the model of the process is contained within the control system. If a control system has its exact model in the control process, then a perfect control system can be obtained (Beni Rehiara et al., 2020; Sahu et al., 2016; Sonker et al., 2019; Vasu et al., 2021). As shown in figure 1.9 of the IMC strategy, the model of process $G_m(s)$ is parallel with the original process $G(s)$. The dissimilarity between the output of the process $Y(s)$ and the output of the model $Y_m(s)$ is known as actual control error (ACE). This error is to be reduced to *'zero'* by using the compensator $Q_c(s)$. The error is generated due to load fluctuations $F_L(s)$ entering into the system. So the IMC-PID controller design aims to reduce the error ACE and equate the process output and model output.

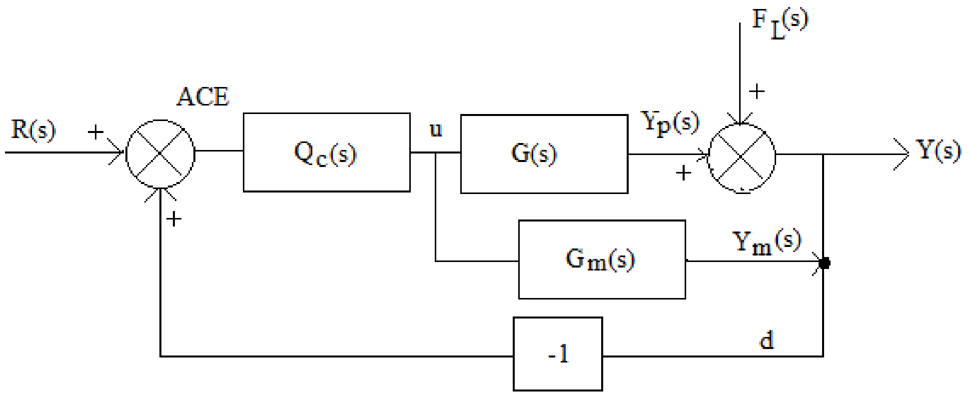


**Figure 1.9. The basic strategy of the IMC technique**

Hence, the designing of the IMC-PID controller involves the knowledge about the following parameters:

a) The transfer function of the process $G(s)$

b) The transfer function of the model $G_m(s)$

c) Process output $Y_p(s)$

d) Model output $Y_m(s)$

e) Set point $R(s)$

f) Actual control error (ACE)

g) Manipulated input to the process *'u'*

h) Load fluctuations $F_L(s)$ or disturbance

i) Estimated disturbance *'d'*

The IMC control strategy to cope with load frequency control is proven to be the best and most reliable control scheme; hence, this scheme is used for the research work conducted.

## 1.12. AREA OF A POWER SYSTEM

A power system is composed of a generator, a turbine and a reservoir. The reservoir is not an essential part of the power system. If the turbine is to reheat, then a reservoir is needed, but if a turbine is of non-reheated type, then no reservoir is required. Depending on these power generation units, a power system is categorized as a thermal or a hydropower system. More than one power generation unit can be interconnected to form a better power generation plant like a hydrothermal power plant, thermal-thermal power plant. The number of control areas in the interconnected power system is termed the particular power system area. Later on, solar and wind power generation units are interconnected with the thermal and hydropower plants and hence increased areas of the power system are achieved. In the thesis, work is done on the power systems containing a single control area, two control areas and three control areas.

## 1.13. NOVELTY AND CONTRIBUTION OF PRESENT RESEARCH WORK

The field of reducing the order of a model is essential to analyse large and complex systems. Many developments have been presented in this area. Still, the limitations of these techniques motivate to work more to develop a universal technique and provide a more accurate approximation of the system of a high order. Therefore, a novel IPCG (Improved Pole Clustering with Genetic algorithm) technique is developed in the thesis that applies to continuous and discrete systems and SISO (Single Input Single Output) and MIMO (Multi Input Multi Output) systems. This technique is a blended form of two techniques: improved pole clustering and genetic optimisation algorithm. The clustering method presented in the thesis is never used with the genetic algorithm in previous works. Moreover, the clustering used here is the new form of clustering approach involving less dominant poles and high dominant poles in forming the equation of denominator multinomial. It signifies that no truncation of poles takes place in the clustering technique presented in the thesis. So, all poles of the system of high order contribute to forming the poles of the system of reduced-order in the proposed IPCG methodology. Furthermore, the performance parameters based on the time and frequency domain are utilised to compare the given and procured systems. The proposed technique is then applied to reduce continuous-time SISO systems up to $48^{th}$ order, discrete-time SISO system up to $8^{th}$ order and continuous-time MIMO system up to 19 order. In the design of the controller, a new combination is formed to draw a two degree of freedom controller. This combination is formed by IMC based controller and a disturbance rejection controller. So, the methodology proposed for reducing the order of the system is found suitable to procure an accurate and matching system of reduced-order for a SISO system of up to $48^{th}$ order and a MIMO system of up to $19^{th}$ order. And the methodology proposed for the design of the controller eliminating the fluctuations in power and frequency is found suitable up to a system of three areas.

## 1.14. ORGANIZATION OF THESIS

The thesis has been started with an introduction to the concept of reducing the order of a system. The need and application of the concept are also justified in the introduction part. Moreover, the techniques that formed the initial phase of the reduction concept are also briefed in chapter 1. Furthermore, the basics and needs of the controllers eliminating the load frequency disturbances are also described there by defining the types of controllers developed so far to remove the problem associated with power systems. The remaining work is distributed in different chapters. The composition of these chapters is described as follows:

**CHAPTER 2:** The next section in the thesis describes the study performed to get the analysis of work carried out in reducing order by different researchers. It forms the basis of the research work carried out in the thesis. The study performed in the literature study involves the work performed by different researchers on the reduction of order and controller design.

**CHAPTER 3:** The study's outcomes in the literature are presented in the research gap chapter. The research gaps help get the objectives of the work, and the objectives to be achieved are described here in chapter 3.

**CHAPTER 4:** The methodology followed to achieve the objectives is shown in chapter 4. The methodology section comprises two parts; the first part presents the methodology proposed to procure the approximated system of reduced-order from the system of high order and the second part presents the proposed method to design the 2-DOF-IMC-PID controller for the removal of power and frequency fluctuations in the power system.

**CHAPTER 5:** The methodology developed in the thesis is then applied on SISO type nine systems of continuous-time and three systems of discrete-time. Moreover, the proposed technique is also applied to the three examples of MIMO type. The accuracy of the system obtained by the proposed method is checked by fetching the error values. Also, the performance of the system of high order and corresponding system of reduced-order is checked and compared with the techniques developed by other researchers. Furthermore, the proposed method of controller design is used to procure the two degrees of freedom controller for a single area, two area and three area power systems.

**CHAPTER 6:** The conclusion of the work performed in the thesis is done in chapter 6. The conclusion is drawn based on the work carried out in chapter 5, showing results obtained after applying the proposed method for reducing the order of a system and designing a controller eliminating the undesired changes in frequency.